There is a certain “formula” or method to doing proofs. Some of the guidelines are given below. The most important factor in learning to do proofs is practice, just as when one is learning a new language.

1) There are very few words needed in the structure of a proof. Organized in rows by synonyms they are:

   To show
   Assume, Let, Suppose, Define, If
   Since, Because, By
   Then, Thus, So
   There exists, There is
   Recall, We know, But.

2) The overall structure of a proof is a block structure like an outline. For example:

   To show: If \( A \) then \( B \) and \( C \).
   Assume: \( A \).
   To show: a) \( B \).
   b) \( C \).
   a) To show: \( B \).
      Thus \( B \).
   b) To show: \( C \).
      Thus \( C \).
   So \( C \).
   So, if \( A \), then \( B \) and \( C \).

3) Any proof or section of proof begins with one of the following
   a) To show: If \( A \) then \( B \).
   b) To show: There exists \( C \) such that \( D \).
   c) To show: \( E \).

   Immediately following this, the next step is

   Case a) Assume the ifs and ‘to show’ the thens. The next lines usually are
      Assume \( A \).
      To show: \( B \).

   Case b) To show an object exists you must find it.
      The next lines usually are
      Define \( xxx \).
      To show: \( xxx \) satisfies \( D \).

   Case c) Rewrite the statement in \( E \) by using a definition
      The next line is usually
      To show \( E’ \).

A useful guideline is, “Don’t think too much.” Following the “method” usually produces a proof without thinking. Most of doing proofs is simply rewriting what has come just before in a different form by plugging in a definition.
There are some kinds of proofs which have a special structure.

**Proofs by contradiction.**

(*) Assume the opposite of what you want to show.

_____.

_____.

End up showing the opposite of some assumption (not necessarily the (*) assumption).

**Contradiction.**

Thus Assumption (*) is wrong and what you want to show is true.

**Counterexamples.**

To show that a statement, “If ___ then ___,” is false you must give an example.

To show: There exists a _____ such that

a) it satisfies the ifs of the statement that you are showing is false.

b) it satisfies the opposite of some assertion in the thens of the statement that you are showing is false.

**Proofs of uniqueness.**

To show that an object is unique you must show that if there are two of them then they are really the same.

To show: A THING is unique.

Assume $X_1$ and $X_2$ are both THINGS.

To show: $X_1 = X_2$.

**Proofs by induction.**

A statement to be proved by induction must have the form

If ___ then ___ for all positive integers $n$

The proof by induction should have the form

**Proof by induction.**

**Base case:**

To show: If ___ then ___ for $n = 1$

_____.

_____.

Thus if ___ then ___ for $n = 1$

**Induction step:**

Assume If ___ then ___ for $n < N$.

To show: If ___ then ___.

This last to show line contains exactly the same statement except with $n$ replaced by $N$ and “for all positive integers $n$” removed.