Problem A. Graphs of the basic functions.

(1) Graph \( f(x) = |x| \).

(2) Graph \( f(x) = \lfloor x \rfloor \).

(3) Graph \( f(x) = 2 \).

(4) Graph \( f(x) = x \).

(5) Graph \( f(x) = x^2 \).

(6) Graph \( f(x) = x^3 \).

(7) Graph \( f(x) = x^4 \).

(8) Graph \( f(x) = x^5 \).

(9) Graph \( f(x) = x^6 \).

(10) Graph \( f(x) = x^{100} \).

(11) Graph \( f(x) = x^{-1} \).

(12) Graph \( f(x) = x^{-2} \).

(13) Graph \( f(x) = x^{-3} \).

(14) Graph \( f(x) = x^{-4} \).

(15) Graph \( f(x) = x^{-100} \).

(16) Graph \( f(x) = e^x \).

(17) Graph \( f(x) = \sin x \).

(18) Graph \( f(x) = \cos x \).

(19) Graph \( f(x) = \tan x \).
(20) Graph \( f(x) = \cot x \).

(21) Graph \( f(x) = \sec x \).

(22) Graph \( f(x) = \csc x \).

(23) Graph \( f(x) = \sqrt{x} \).

(24) Graph \( f(x) = x^{1/3} \).

(25) Graph \( f(x) = x^{1/4} \).

(26) Graph \( f(x) = x^{1/5} \).

(27) Graph \( f(x) = x^{1/6} \).

(28) Graph \( f(x) = \frac{1}{\sqrt{x}} \).

(29) Graph \( f(x) = x^{-1/3} \).

(30) Graph \( f(x) = x^{-1/4} \).

(31) Graph \( f(x) = \ln x \).

(32) Graph \( f(x) = \sin^{-1} x \).

(33) Graph \( f(x) = \cos^{-1} x \).

(34) Graph \( f(x) = \tan^{-1} x \).

(35) Graph \( f(x) = \cot^{-1} x \).

(36) Graph \( f(x) = \sec^{-1} x \).

(37) Graph \( f(x) = \csc^{-1} x \).

**Problem B. Where is a function continuous?**

(1) For which values of \( x \) is the function \( f(x) = x^2 + 3x + 4 \) continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
(2) For which values of $x$ is the function $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(3) For which values of $x$ is the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(4) For which values of $x$ is the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(5) Determine the value of $k$ for which the function $f(x) = \begin{cases} \frac{\sin 2x}{5x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0, \end{cases}$ is continuous at $x = 0$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(6) For which values of $x$ is the function $f(x) = \begin{cases} x - 1, & \text{if } 1 \leq x < 2, \\ 2x - 3, & \text{if } 2 \leq x \leq 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(7) For which values of $x$ is the function $f(x) = \begin{cases} \cos x, & \text{if } x \geq 0, \\ -\cos x, & \text{if } x < 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(8) For which values of $x$ is the function $f(x) = \begin{cases} \sin(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(9) Find the value of $a$ for which the function $f(x) = \begin{cases} ax + 5, & \text{if } x \leq 2, \\ x - 1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(10) For which values of $x$ is the function $f(x) = \begin{cases} 1 + x^2, & \text{if } 0 \leq x \leq 1, \\ 2 - x, & \text{if } x > 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
(11) For which values of $x$ is the function $f(x) = 2x - |x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(12) Find the value of $a$ for which the function $f(x) = \begin{cases} 2x - 1, & \text{if } x < 2, \\ a, & \text{if } x = 2, \\ x + 1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(13) For which values of $x$ is the function $f(x) = \begin{cases} \frac{|x - a|}{x - a}, & \text{if } x \neq a, \\ 1, & \text{if } x = a, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(14) For which values of $x$ is the function $f(x) = \begin{cases} \frac{x - |x|}{2}, & \text{if } x \neq 0, \\ 2, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(15) For which values of $x$ is the function $f(x) = \begin{cases} \sin x, & \text{if } x < 0, \\ x, & \text{if } x \geq 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(16) For which values of $x$ is the function $f(x) = \begin{cases} \frac{x^n - 1}{x - 1}, & \text{if } x \neq 1, \\ n, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(17) Explain how you know that $f(x) = \sec x$ is continuous for all values of $x$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(18) For which values of $x$ is the function $f(x) = \cos |x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(19) For which values of $x$ is the function $f(x) = [x]$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

(20) For which values of $x$ is the function $f(x) = \begin{cases} x^3 - x^2 + 2x - 2, & \text{if } x \neq 1, \\ 4, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
(21) For which values of $x$ is the function $f(x) = |x| + |x - 1|, -1 \leq x \leq 2$, continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

**Problem C. Existence of limits.**

(1) Explain why \( \lim_{x \to 0} \frac{1}{x} \) does not exist.

(2) Explain why \( \lim_{x \to \pi/2} \tan x \) does not exist.

(3) Explain why \( \lim_{x \to \pi/2} \sec x \) does not exist.

(4) Explain why \( \lim_{x \to 0} \csc x \) does not exist.

(5) Explain why \( \lim_{x \to -1} \ln x \) does not exist.

(6) Explain why \( \lim_{x \to 0} \sin(1/x) \) does not exist.

(7) Explain why \( \lim_{x \to \infty} \text{sgn}(x) \) does not exist.

(8) Let \( \text{sgn}(x) \) be the sign function. This function is given by \( \text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases} \) Explain why \( \lim_{x \to \infty} \text{sgn}(x) \) does not exist.

(9) Explain why \( \lim_{x \to 0} 2^{1/x} \) does not exist.

(10) Explain why \( \lim_{x \to 1} 2^{1/(x-1)} \) does not exist.

**Problem D. Increasing, decreasing, and concavity.**

(1) What does it mean for a function \( f(x) \) to be continuous at \( x = a \)? Explain how to test if a function is continuous at \( x = a \).

(2) What does it mean for a function \( f(x) \) to be differentiable at \( x = a \)? Explain how to test if a function is differentiable at \( x = a \).

(3) What does \( \frac{df}{dx} \bigg|_{x=a} \) indicate you about the graph of \( y = f(x) \)? Explain why this is true.
(4) What does it mean for a function to be increasing? Explain how to use calculus to tell if a function is increasing. Explain why this works.

(5) What does it mean for a function to be concave up? Explain how to use calculus to tell if a function is concave up. Explain why this works.

(6) What is a critical point? Explain how to find critical points of a function $f(x)$?

(7) What is a point of inflection? Explain how to find points of inflection of a function $f(x)$?

(8) What is an asymptote of a function $f(x)$? Explain how to justify that a given line is an asymptote of $f(x)$?

(9) If $f(x) = |x|$ what is $\frac{df}{dx}|_{x=2}$?

(10) Find the values of $a$ and $b$ so that the function $f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1, \\ bx + 2, & \text{if } x > 1, \end{cases}$ is differentiable for all values of $x$.

**Problem E. Graphing polynomials.**

For each of the following graphing problems also determine

(a) where $f(x)$ is defined,
(b) where $f(x)$ is continuous,
(c) where $f(x)$ is differentiable,
(d) where $f(x)$ is increasing and where it is decreasing,
(e) where $f(x)$ is concave up and where it is concave down,
(f) what the critical points of $f(x)$ are,
(g) where the points of inflection are, and
(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

(1) Graph $f(x) = a$, where $a$ is a constant.

(2) Graph $f(x) = ax + b$, where $a$ and $b$ are constants.

(3) Graph $f(x) = a(x - c) + b$, where $a$, $b$ and $c$ are constants.

(4) Graph $f(x) = \begin{cases} 2 - x, & \text{if } x \geq 1, \\ x, & \text{if } 0 \leq x \leq 1. \end{cases}$
(5) Graph \( f(x) = \begin{cases} 2 + x, & \text{if } x \geq 0, \\ 2 - x, & \text{if } x < 0. \end{cases} \)

(6) Graph \( f(x) = \begin{cases} 1 - x, & \text{if } x < 1, \\ x^2 - 1, & \text{if } x \geq 1. \end{cases} \)

(7) Graph \( f(x) = 2x - x^2. \)

(8) Graph \( f(x) = x - x^2 - 27. \)

(9) Graph \( f(x) = 3x^2 - 2x - 1. \)

(10) Graph \( f(x) = x^3. \)

(11) Graph \( f(x) = x^3 - x + 1. \)

(12) Graph \( f(x) = x^3 - x - 1. \)

(13) Graph \( f(x) = (x - 2)^2(x - 1). \)

(14) Graph \( f(x) = 2x^3 - 21x^2 + 36x - 20. \)

(15) Graph \( f(x) = 2x^3 + x^2 - 20x. \)

(16) Graph \( f(x) = 1 - x^4. \)

(17) Graph \( f(x) = 3x^4 - 4x^3 - 12x^2 + 5. \)

(18) Graph \( f(x) = 3x^4 - 16x^3 + 18x^2. \)

(19) Graph \( f(x) = x^5 - 4x^4 + 4x^3. \)

(20) Graph \( f(x) = x^3(x - 2)^2. \)

(21) Graph \( f(x) = (x - 2)^4(x + 1)^3(x - 1). \)