MATH 221 lecture 2, September 8, 2000

Angles

It is the distance halfway around a circle of radius 1.

Measure angles according to the distance traveled on a circle of radius 1.

The angle $\theta$ is measured by traveling a distance $\theta$ on a circle of radius 1.

Stretch both $x$ and $y$ to get a circle of radius $r$.

The distance $\theta$ stretches to $r\theta$.

The distance for around a circle of radius 1 stretches to $2\pi r$ around a circle of radius $r$.

So the circumference of a circle is $2\pi r$, if the circle is radius $r$.

To find the area of a circle first approximate with a polygon inscribed in the circle.

The eight triangles form an octagon $P_8$ in the circle. The area of the octagon $P_8$ is almost the same as the area of the circle.

Unwrap the octagon.

The area of the octagon is the area of the 8 triangles. The area of each triangle is $\frac{1}{2}bh$.

So the area of the octagon is $8 \times \frac{1}{2}bh$.

Take the limit as the number of triangles in the interior polygon gets larger and larger (the polygon gets closer and closer to being the circle). Then
Area of the circle = \lim_{n \to \infty} \frac{1}{n} \pi \text{ (area of an n-sided polygon P)}

= \lim_{n \to \infty} \left( \frac{1}{2} \text{Bh} \right)
= \frac{1}{2} \text{ length of an unrolled circle}
= \frac{1}{2} (2\pi r)(r)
= \pi r^2.

So the area of a circle is \pi r^2 if the circle is radius r.

Trigonometric functions

\sin \theta is the y-coordinate of a point at distance \theta on a circle of radius 1.
\cos \theta is the x-coordinate of a point at distance \theta on a circle of radius 1.

\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \csc \theta = \frac{1}{\sin \theta}.

Since the equation of a circle of radius 1 is \(x^2 + y^2 = 1\), this forces \sin^2 \theta + \cos^2 \theta = 1.

The pictures show that \sin(-\theta) = -\sin \theta and \cos(-\theta) = \cos \theta.

Also, \sin \frac{\pi}{2} = 1 and \cos \frac{\pi}{2} = 0.

Draw the graphs of \(y = \sin \theta\) and \(y = \cos \theta\).
by seeing how the $x$ and $y$ coordinates change as you walk around the circle.

Example: Verify $\sec B - \tan B = 1$.

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\frac{\sec B - \tan B}{\cos B \cot B} = \frac{1}{\cos B} - \frac{\sin B}{\cos B} = \frac{1 - \sin^2 B}{\cos^2 B} = \frac{\cos^2 B}{\cos^2 B} = 1.
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Example: Verify $\cot x - \cot y = \frac{\sin(y-x)}{\sin x \sin y}$

Left Hand Side: $\cot x - \cot y = \frac{\cos x}{\sin x} - \frac{\cos y}{\sin y}$

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= \frac{\cos x \sin y - \cos y \sin x}{\sin x \sin y}.
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Right Hand Side: $\frac{\sin(y-x)}{\sin x \sin y} = \frac{\sin y \cos x - \cos y \sin x}{\sin x \sin y}$

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= \frac{\sin x \cos x - \cos x \sin x}{\sin x \sin y} = \frac{\sin x \cos x - \cos x \sin x}{\sin x \sin y}.
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\text{Left Hand Side} = \text{Right Hand Side}.
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