

# Semi-decentralised Coordination of the Charging of Electric Vehicles

Student: Ryan Tan

Supervisor: Matthew Tam

## Introduction

To reduce greenhouse gas emissions, many people have switched from using fossil fuel vehicles over to plug-in electric vehicles (PEVs). However, this brings upon a new dilemma: each person will make individually optimal charging decisions (such as minimising electricity costs).

This is not globally optimal for the system, as their decisions are uncoordinated and potentially at odds with the power grid's goal of balancing power across time, resulting in unnecessary stress on the power grid.

My project involves modelling the vehicles as agents in an aggregative game, in order to reduce the stress on the power grid.

I will focus on the Forward-Backward algorithm implementation, but many other algorithm designs exist, such as the Forward-Reflected-Backward and Forward-Backward-Forward.

## Electric Vehicle Modelling

Let  $x_i \in \mathbb{R}^n$  represent the the charging decisions chosen by each agent  $i \in \mathcal{I} := \{1, 2, \dots, N\}$  across a time horizon of  $n = 24$  hours. Each agent is allowed to select charging decision from its local decision set  $\Omega_i$ . To keep notation light,  $\mathbf{x} := \text{col}(x_1, \dots, x_i, \dots, x_N)$  and  $\mathbf{x}_{-i} = \text{col}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N)$ .

Modelling this problem as an aggregative game gives a system of inter-dependent optimisation problems, where  $\forall i \in \mathcal{I}$ :

$$\begin{cases} \underset{x_i \in \mathbb{R}^n}{\text{argmin}} & J_i(x_i, \mathbf{x}_{-i}) := g_i(x_i) + p(\text{avg}(\mathbf{x}))^T x_i & (1a) \\ \text{s.t.} & x_i \in \Omega_i & (1b) \\ & \sum_{i=1}^N x_i(t) \leq NK(t), \forall t = 1, \dots, n & (1c) \end{cases}$$

**The Cost Function** - (1a) the  $g_i$  term models the individual costs of the agent, such as battery degradation, or penalty when the agent does not adhere to a preferred charging strategy. The  $p(\text{avg}(\mathbf{x}))^T x_i$  term models the energy price, and contains information such as the non-electric-vehicle energy demand (Fig. 1).

**The Local Decision set** - (1b) models the decisions available to the agent.

**The Coupling Constraint** - (1c) models the max power the grid can deliver to the electric vehicles (for a given time).

The optimisation problem can be characterised in terms of the Karush-Kuhn-Tucker conditions (a generalisation of the method of Lagrange multipliers). Solving this is equivalent to finding the zeros of an operator. Then using techniques of monotone operator splitting, Algorithm 1 can be formed. Solving the optimisation problem (1) would mean obtaining  $x_i^*$ , for all  $i \in \mathcal{I}$ , such that:

$$J_i(x_i^*, \mathbf{x}_{-i}^*) \leq \inf\{J_i(y, \mathbf{x}_{-i}^*) \mid y \in \mathcal{X}_i(\mathbf{x}_{-i}^*)\}$$

Such solutions are known as a generalised Nash equilibrium (GNE). With  $\mathcal{X}_i(\mathbf{x}_{-i}^*)$  being the set of *feasible* decisions for each agent, meaning the set contains decisions  $x_i$  which satisfy both (1b) and (1c).

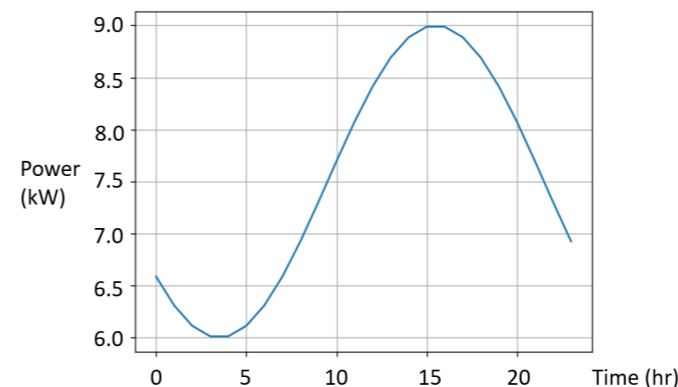


Figure 1: Approximate non-electric-vehicle energy demand across 24hrs, modelled as a sinusoidal function, with peak power demand occurring at about 3pm

## Research Experience

Despite not being able to be in person at the University to do this project, I still found this to be a fun and enjoyable experience. Learning how to extract the important information, and diving further into the other papers referenced, was quite an adventure.

I also had the great opportunity to attend talks from conferences such as WoMBaT and AustMS, and gained experience giving a short talk about my research to the other students under my supervisor.

## Algorithm 1

Preconditioned Forward-Backward (pFB)

**Initialisation:**  $\delta > \frac{1}{2\gamma}; \forall i \in \mathcal{I}, x_i^0 \in \mathbb{R}^n$ ,

$0 < \alpha_i \leq (\|I_n\| + \delta)^{-1}; \lambda^0 \in \mathbb{R}_{\geq 0}^n$

$0 < \beta \leq (\frac{1}{N} \sum_{i=1}^N \|I_n\| + \frac{1}{N} \delta)^{-1}$

**Iterate until convergence:**

1. Local: Strategy update, for all  $i \in \mathcal{I}$ :

$$y_i^k = x_i^k - \alpha_i [\nabla_{x_i} p(\text{avg}(\mathbf{x}))^T x_i + \lambda^k]$$

$$x_i^{k+1} = \text{prox}_{\alpha_i g_i + \iota_{\Omega_i}}(y_i^k)$$

$$d_i^{k+1} = 2x_i^{k+1} - x_i^k - K$$

2. Central coordinator: dual variable update

$$\lambda^{k+1} = \text{proj}_{\mathbb{R}_{\geq 0}^n}(\lambda^k + \beta \text{avg}(\mathbf{d}^{k+1}))$$

## Graphical Results:

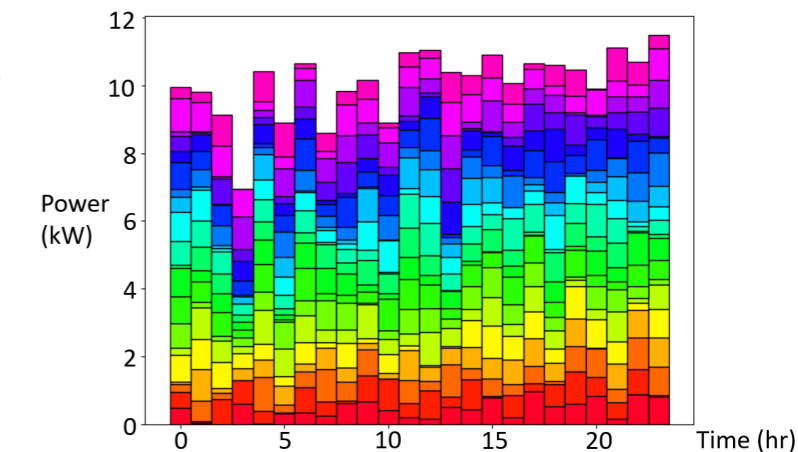


Figure 2: Initial random decisions of 20 agents, each colour represents a unique agent's charging decision across the 24 hours

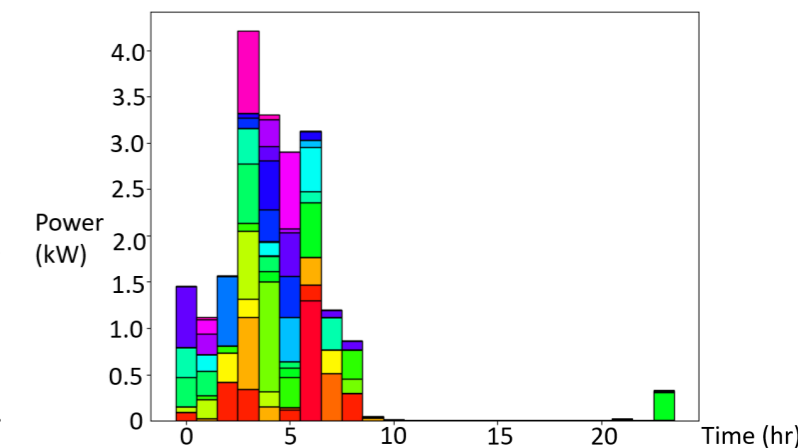


Figure 3: Final converged decisions, notice how this graph opposes Fig 1, in an attempt to evenly distribute power across time. Convergence considered when:

$$\|\text{col}(\mathbf{x}^k, \lambda^k) - \text{col}(\mathbf{x}^{k+1}, \lambda^{k+1})\| < 10^{-5}$$

<sup>1</sup>CAJC: in the PNW. *More Power*. Flickr. Used under CC BY-NC-ND 2.0. Title layered on original.