MUMS Problem Solving Competition

Melbourne University Mathematics and Statistics Society

25 June, 2014

RULES

- Teams must have either three or four competitors.

- The competition will consist of five rounds, each lasting ten minutes.

- In each round there are five questions, but only your *best three* questions will contribute to your score. The questions have 3, 4, 5, 6 and 7 points allocated respectively.

- You will not lose points for incorrect answers.

- Prizes will be awarded to the top three school teams.

- You must have fun!

ROUND ONE

1. What is the largest odd factor of 2014?

2. Jinghan loves to bake dodgy cakes (true story). She makes fairy cake with 24 cm^3 of self raising flour, 20 cm^3 of egg, 20 cm^3 of sugar and 12 cm^3 of milk. If the flour rises by 50%, the egg rises by 20% and 25% of the milk evaporates. How big is the cake after it is baked?

3. What is the largest 3 digit number such that the sum of its digits is equal to the square of its last digit?

4. Bobby Blue Fairy has decided to make his own fairy bread. The ratios of sweetness to awesomeness to magicness are as follows. Blue 3:4:1, pink 0:0:1, yellow 2:1:1. What ratio of blue to pink to yellow sprinkles should Bobby blue fairy use so that the sprinkle mixture has equal parts sweetness, awesomeness and magicness?

5. The ages of Ann and Bill add to 91. Ann is now 3 times older than Bill was at the time when Ann was twice as old as Bill is now. How old are Ann and Bill?

ROUND TWO

1. Declan decides to roll down from the top of a hill in a ball with radius 1 m. If the length of the slope is 10π m, how many rotations has the ball made when it reaches the bottom?

2. What is the smallest 3 digit number so that when you write its digits backwards, the value of the new number increases by more than a factor of 2?

3. Jinghan has a very cheap dodgy toaster (true story). She rates the darkness of her toast from 0–5, where 0 is untoasted and 5 is totally burnt. The left slot cooks the left side at 1 degree of darkness per minute and the right side at 1.5 degrees of darkness per minute. The right slot cooks the left side at 3 degrees of darkness per minute and the right side at 0.5

degrees of darkness per minute. For how long must Jinghan put her bread in the left and then (without flipping) the right slot of the toaster so that both sides are toasted to a darkness of 4?

4. Put the numbers 0,1,2,3,4 and 5 into the circles so that every pair of circles which are joined by a straight line contain two numbers which add up to a prime number.
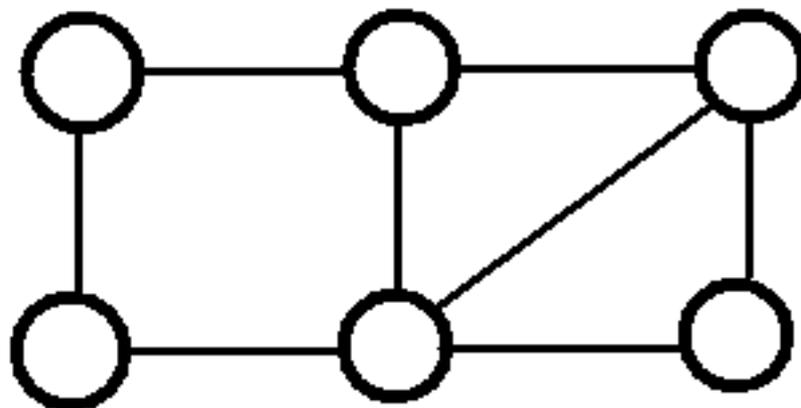

5. Calculate the following sum:

$$6 + 66 + 666 + 6666 + 66666 + 666666$$

ROUND THREE

1. Mel talks at a rate of 20 words per minute, Jinghan talks at a rate of 70 words per minute (true story). They have a conversation for 5 minutes in which 250 words were said. How long did Mel talk for?

2. Damian has two clocks, one with (hour and minute) hands which travel in a clockwise direction, and another with hands which travel in an anti-clockwise direction. Assuming the hands are in the same position at 12 o'clock, how many times a day are the hands of the two clocks in the same position?

3. Yi swims across a river which is 50m wide and then swims back. He swims at 2m/s orthogonal to the 5m/s current. How far downstream would he be when he gets back?

4. Combine plus signs and five 2s to get 28, and combine plus signs and eight 8s to get 1000.

5. How many paths are there from A to B if you are only allowed to travel to the right on the grid?
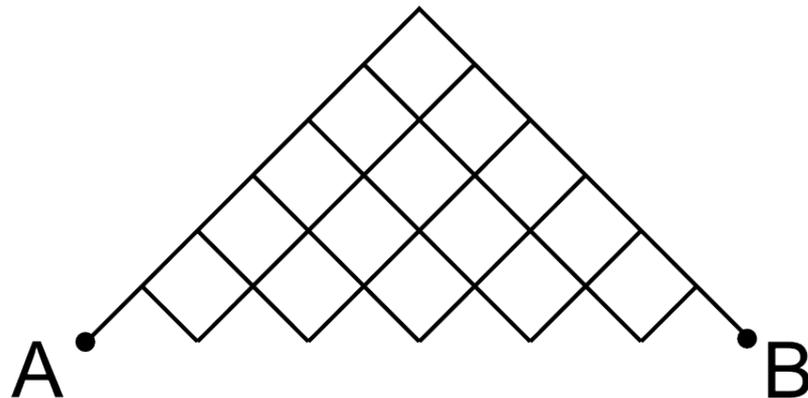
ROUND FOUR

1. Bobby Blue Fairy loves the blue sprinkles on fairy bread the best. Sarah makes fairy bread with sprinkles that are 1 part blue, 2 parts yellow, and 3 parts pink and spreads 30 sprinkles/cm^2. Paul makes fairy bread with sprinkles that are 3 parts blue, and 2 parts white and spreads 10 sprinkles/cm^2. Ben makes fairy bread with sprinkles that are 3 parts blue, 3 parts pink, and 2 parts brown and spreads 20 sprinkles/cm^2. If they all use the same sized bread, who should Bobby Blue fairy gets his fairy bread from to have the most blue sprinkles?

2. Tree Trunks bakes a delicious ruby-red apple pie. Finn the human eats a 135° slice of the pie, Jake the dog eats a 180° slice of the pie, Princess Bubblegum eats a 30° slice and Marceline sucks up all of

the red-colour of the pie but eats none. What fraction of Tree Trunks's pie is left?

3. Today is Wednesday 25 June. What will be the next year in which 25 June is a Wednesday?

4. Three different positive whole numbers add up to 9. What is the maximum possible value of their product?

5. Take two equilateral triangles $\triangle ABC$ and $\triangle DEF$ of side-length 1 and glue vertices A and F together. What is the largest possible area of the quadrilateral $BCDE$?
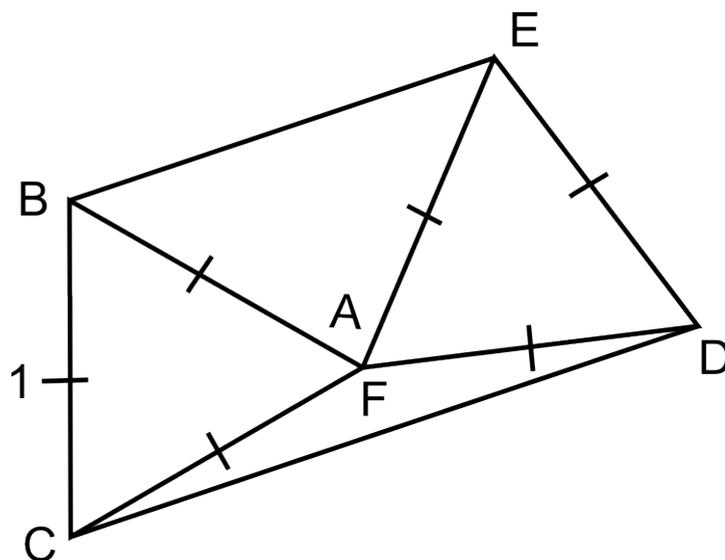
ROUND FIVE

1. Yi and Thara are competing in a 100m sprint. Yi finishes half a second faster than Thara. If their total time is 21.5 seconds, what was Thara's time?

2. Bobby Blue Fairy is having a party in a room shaped like the picture below. He wants to put plates of fairy bread around the room so each square either contains or is adjacent to a square that contains a plate of fairy bread. (Diagonally next to doesn't count.) What is the fewest number of plates of fairy bread that Bobby Blue Fairy has to prepare for his party?
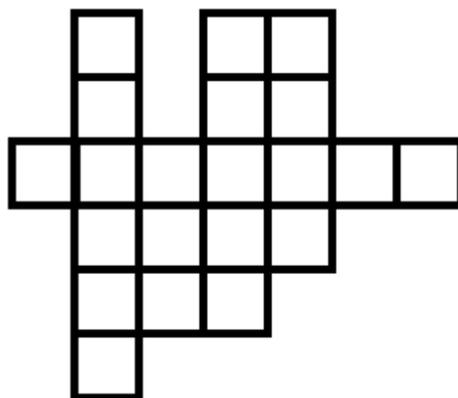
3. Which two distinct integers satisfy the equation: $a^b = b^a$

4. In how many ways can you shade two squares on a 4×4 grid such that they share exactly one corner with each other?

5. Naruto and Sakura (a couple) went to a business meeting with two other couples: Ash & Misty (a couple), and Spiderman & Mary Jane Watson (another couple). Several handshakes took place. No one shook hands with themselves or their partner and no one shook hands with the same person more than once. Naruto needs your help, he forgot how many hands he shook. He asked each person, including Sakura, how many hands they shook. Each person gave a different answer. How many hands did Naruto shake?

TIE-BREAKER

1. Apple, Blanket, Cricket and Destry meet up once weekly for paparazzi-playdates. After consulting with a numerologist (and a seance for Blanket's father), their parents decide that each week:

   - 1 of the kids must hi-five precisely 3 other children,

   - 2 of the kids must hi-five precisely 2 other children,

   - 1 of the kids must hi-five precisely 1 other child,

   - and the hi-five configuration must be different each week.

   What is the maximum number of weeks for which these playdates can last?