# Trajectory of magnetotactic bacteria under the influence of magnetic field

Xinyi Yang with supervisor Douglas Brumley

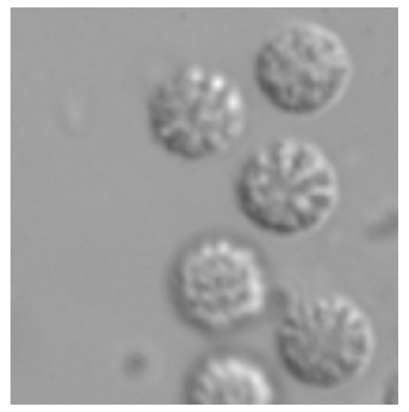
University of Melbourne

## Introduction

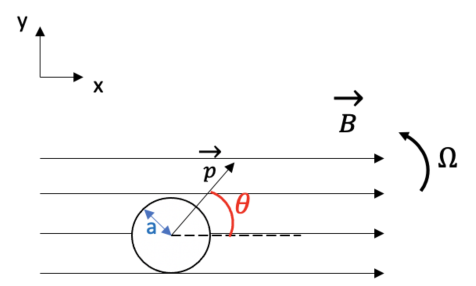
Magnetotactic bacteria (MTB) orient themselves to align with magnetic field like a compass needle. Here we study the swimming trajectory of a single MTB under the influence of magnetic field.

## Model

- Assume the bacteria has magnetic moment $\vec{m}$ which is parallel to its orientation.
- The particle swims at a constant speed v, in the direction of its orientation vector.
- We assume the Reynolds number in our problem is zero.
- The magnetic field rotates anticlockwise in a constant angular speed $\Omega$.



(a) An image of MTB captured in the Brumley lab.




(b) A diagram illustrating the system.

Governing equations:

$$\begin{cases} \dfrac{d\vec{r}}{dt} = v\vec{p} \\ \vec{m} \times \vec{B} + \vec{\tau}_{drag} = \vec{0} \end{cases}$$

where $\vec{\tau}_{drag} = -8\pi\mu a^3\vec{\omega}$ and $\vec{r}$ is the position vector of the bacteria.

## Stationary uniform magnetic field

This is the special case where $\Omega = 0$ and we are able to obtain an analytical solution for $\vec{r}$ :

$$\vec{r}(t) = (r_x(t), r_y(t)) = \left( vt - \frac{v}{k}\log\left(\frac{1+l^2}{1+l^2e^{-2kt}}\right), \frac{v\alpha}{k} - \frac{2v}{k}\arctan\left(le^{-kt}\right) \right)$$

,where k = $\frac{M\|\vec{B}\|}{8\pi\mu a^3}$, and the unit of k is $s^{-1}$, which can be understood as the alignment rate of the bacteria to the magnetic field.

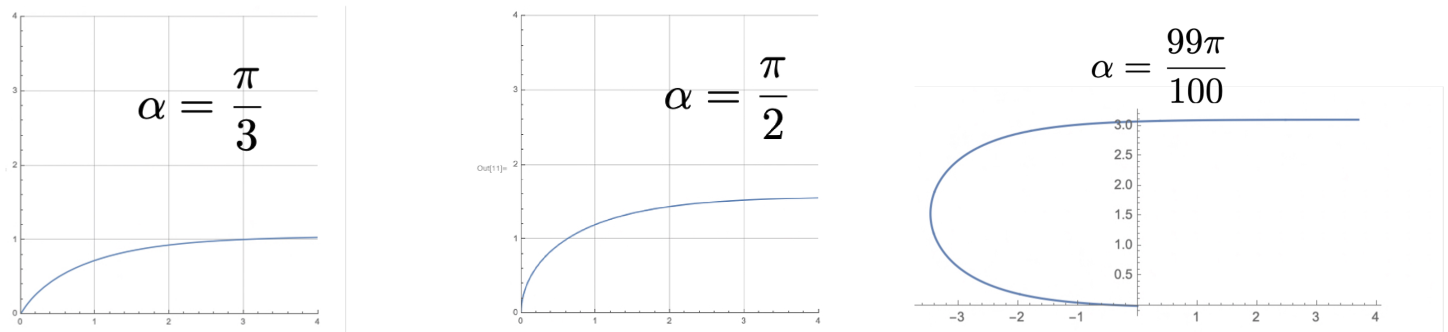



Figure 2. Typical trajectories under different initial conditions.

## Time and length scale of trajectory

It turns out that the time and length scale of the trajectory is controlled by some parameters.

- For stationary uniform magnetic field,

$$\theta(bk, \frac{t}{b}) = \theta(k, t)$$

This means for **different** k values, the geometry of the trajectory won't vary. However the length scale and the time scale of the trajectory will change accordingly.

- For rotating uniform magnetic field,

$$\theta(b\Omega, bk, \frac{t}{b}) = \theta(\Omega, k, t)$$

This means for **same** $\frac{\Omega}{k}$ ratio, the geometry of the trajectory won't vary, given we look at it on a different time and length scale.

**This is important since it tells us that $\frac{\Omega}{k}$ determines the geometry of the trajectory!**

## Rotating magnetic field

This is when the magnetic field rotates with a constant angular speed, and the dynamic of the system is characterised by $\frac{\Omega}{k}$, which we will discuss separately. $k = \frac{M\|\vec{B}\|}{8\pi\mu a^3}$ is proportional to the magnetic field strength.

### $\frac{\Omega}{k} \leq 1$

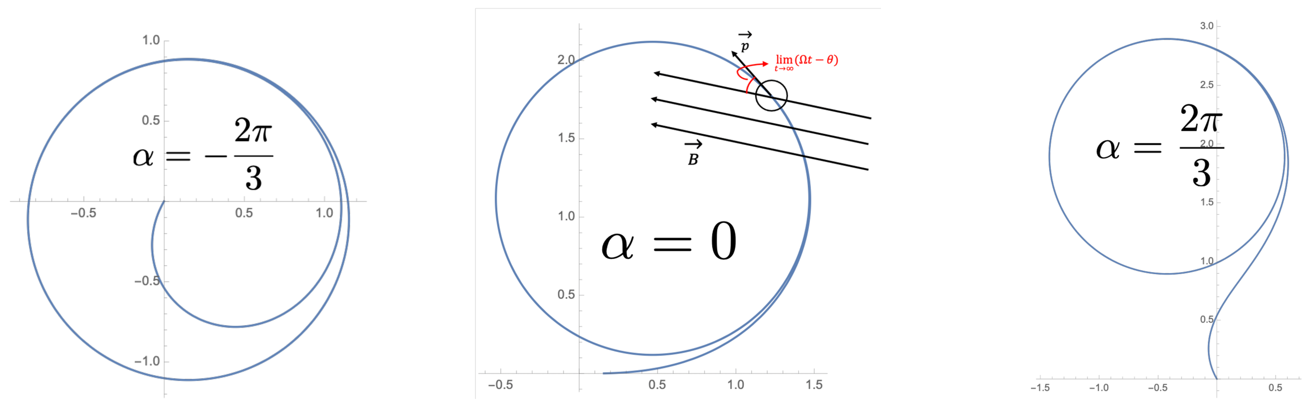
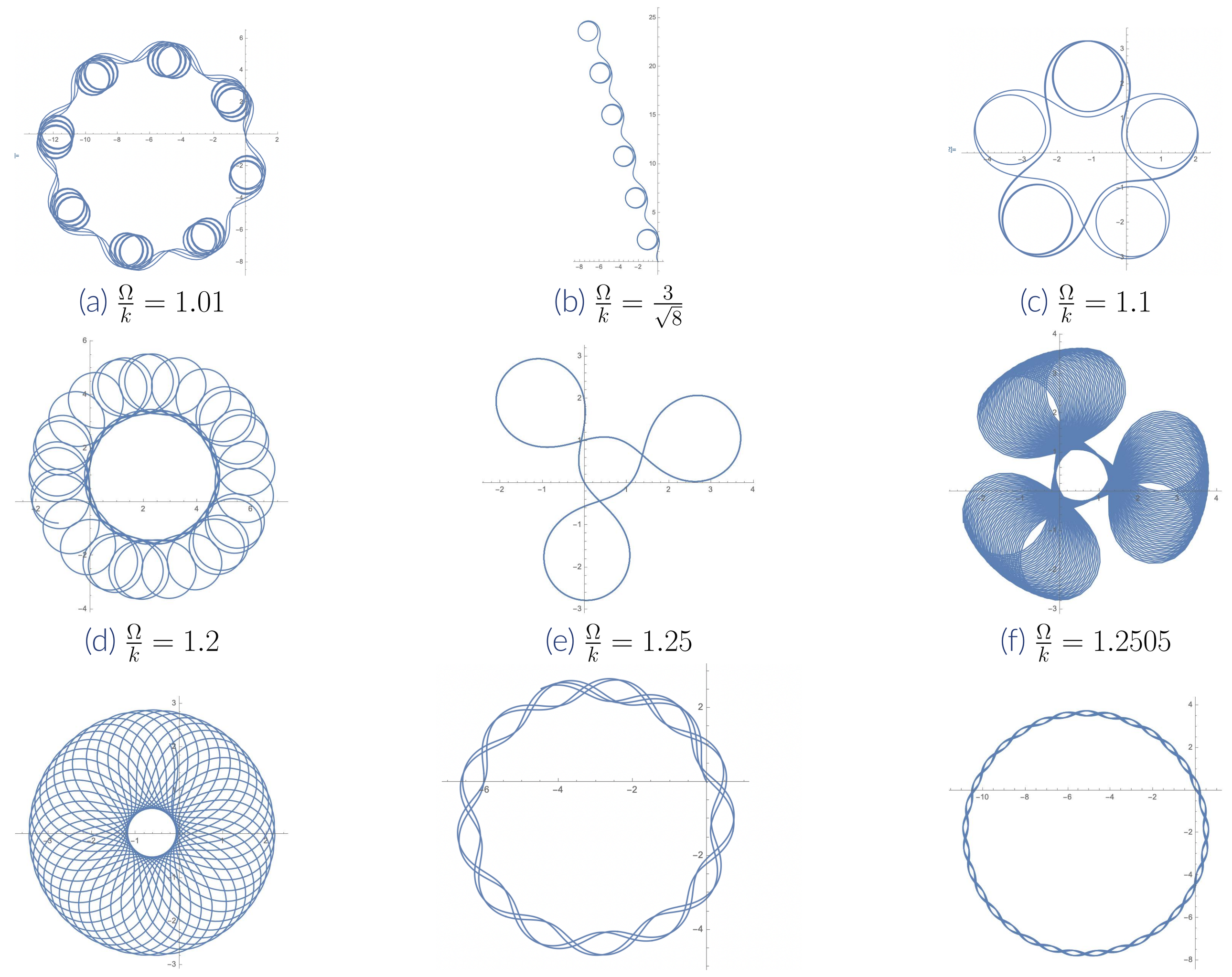



Figure 3. Typical trajectories under different initial conditions when $\Omega \leq k$.

- For $\frac{\Omega}{k} \leq 1$, the bacteria is able to catch up with the magnetic field, and keeps a **stable angle** between its orientation and the direction of magnetic field as shown in (b). The magnetic field then applies a constant torque on the bacteria, causing it to swim in a stable circular trajectory. The radius of the stable circular trajectory is given by $\mathbf{r} \approx \frac{\mathbf{v}}{\mathbf{\Omega}}$.
- When $\frac{\Omega}{k} = 1$, the stable angle is $\frac{\pi}{2}$. This is when the magnetic field can just keep the bacteria in a stable circular trajectory. If $\Omega$ is greater than $k$, then the stable circular trajectory is ruined, which leads us to the case where $\frac{\Omega}{k} > 1$.

### $\frac{\Omega}{k} > 1$



(a) $\frac{\Omega}{k} = 1.01$ (b) $\frac{\Omega}{k} = \frac{3}{\sqrt{8}}$ (c) $\frac{\Omega}{k} = 1.1$
(d) $\frac{\Omega}{k} = 1.2$ (e) $\frac{\Omega}{k} = 1.25$ (f) $\frac{\Omega}{k} = 1.2505$
(g) $\frac{\Omega}{k} = 1.35$ (h) $\frac{\Omega}{k} = 2$ (i) $\frac{\Omega}{k} = 3$

Figure 4. Some example trajectories when $\Omega > k$.

It was discovered that the particle will **retrace** its trajectory **if and only if** the following equality is satisfied:

$$\frac{\Omega}{k} = \frac{n_r}{\sqrt{n_r^2 - n_p^2}}$$

- If $n_p = 1$, then $\frac{\Omega}{k}$ satisfying the equality describes trajectories that will progress in a linear fashion, like the one shown in Figure 4(b).
- If $n_p \neq 1$, then $\frac{\Omega}{k}$ satisfying the equality describes trajectories that retrace themselves, like the one shown in Figure 4(e).

## Stochastic trajectories

- Explore the trajectory of MTB in a more realistic setting by introuding a noise term.
- Simulation using RK4 method with step size h = 0.01 second.
- Control the geometry of the determinisitc trajectory (i.e. keep $\frac{\Omega}{k}$ fixed).
- Try to understand how the magnetic field and the viscosity of fluid affect the stochastic trajectory.

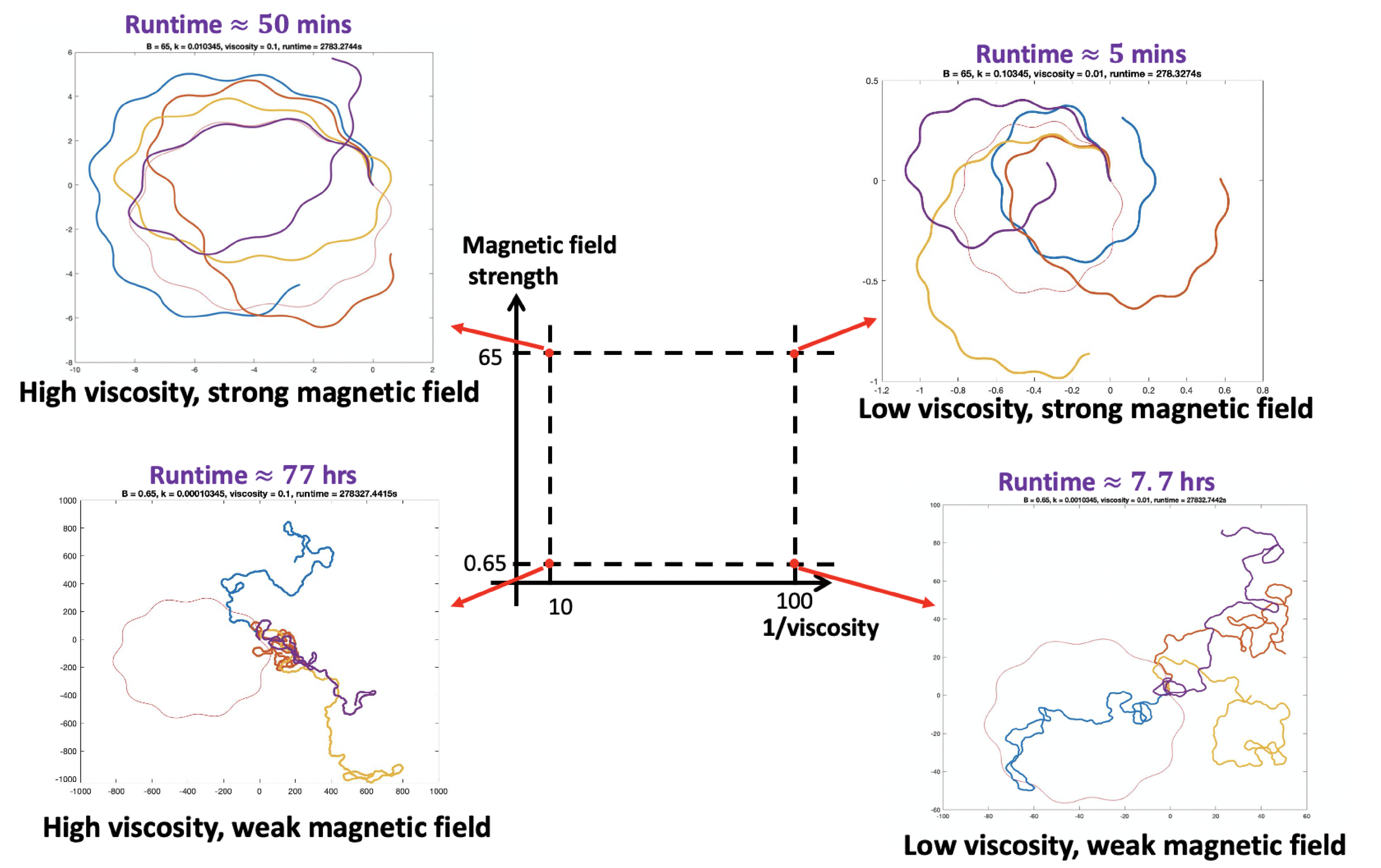



Figure 5. Simulated stochastic trajectories under extreme values of viscosity and magnetic field strength. The closed polygon-like trajectory is the deterministic trajectory under same conditions to be compared against the stochastic trajectories.

Inspired by the above figure, we investigate two situations and developed the following understanding:

1. For a fixed magnetic field strength, changing viscosity would result in **same stochastic behaviour on a different time and length scale**. See the left figure below.
2. On the other hand, for a fixed viscosity, a stronger magnetic field would result in a **less deviated stochastic trajectory**. See the right figure below.

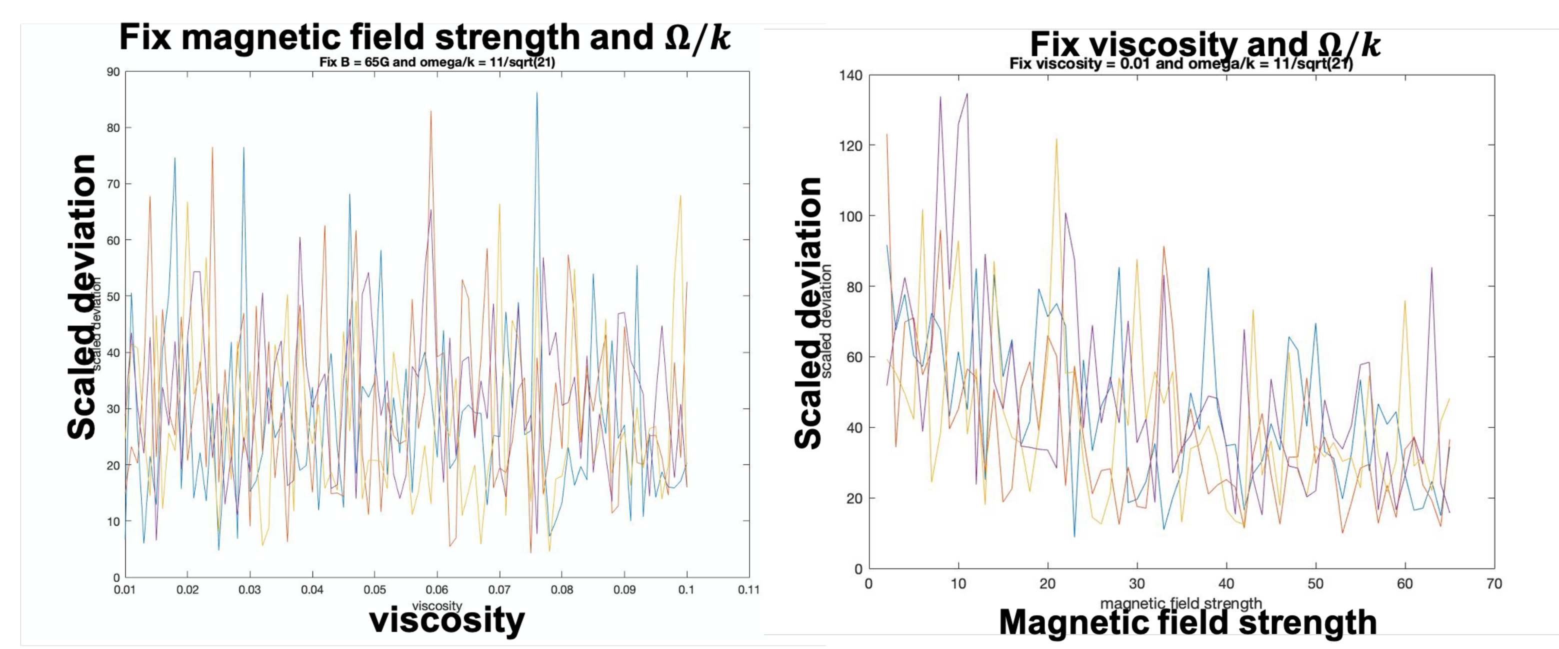



Figure 6. Left figure shows the effect of viscosity on the stochastic trajectory when magnetic field strength and $\frac{\Omega}{k}$ are fixed. Right figure shows the effect of magnetic field strenght on the stochastic trajectory when viscosity and $\frac{\Omega}{k}$ are fixed.

## Acknowledgements

I would like to say a big thank you to my supervisor Douglas for his generous support and patience. I appreciate the freedom I was given to explore different ideas, while feeling well guided whenever I get stuck on a problem. Also thanks to the Department of Maths and Stats for giving me the opportunity to have this unforgettable experience.