Tian Sang—Second Year BSc

The summer of 2011 should be considered as one of the most enjoyable summers I’ve ever had. I was so grateful for having the golden opportunity to work on a vacation scholarship project with my supervisor Dr. Lawrence Reeves.

**Poisson Boundary of Random Walks**

My project involved studying the Poisson boundary of hyperbolic groups. The “Poisson boundary” is the probability space of the events in the paths with respect to the time shift of the associated random walk for the group.

Intuitively, we have a group $G$ with probability measure $\mu$ on $G$, $\mu$ has finite first moment. Then let $x_1, x_2, x_3, \ldots$ be a sequence of independent random variables. The sequence $\{y_n\}$ where $y_n = x_1, x_2, \ldots, x_n$ is the random walk induced by $\mu$. Associated to this walk is a probability space $(\Gamma, \nu)$ is called the Poisson boundary.

**Lemma and Results**

We particularly looked at the proof of one lemma from Danny Calegari’s paper to understand the long term behaviour of a sequence in the group related to a geodesic ray.

This lemma demonstrated that in a hyperbolic space, how a sequence in the group can behave close enough to the geodesic ray in long term, and therefore converges to a point on the boundary of the hyperbolic group.

An important result of Poisson boundary of hyperbolic group is that it captures the random walk’s asymptotic behaviour. We can think of fixing a base point in the group, then start the random walk associated with certain measure $\mu$ from that base point. After passing a certain number of elements, we will reach to the geometric boundary of the group eventually. The Poisson boundary describe how the random walks starting from base point converge to the geometric boundary of the group.

**Experience**

The vacation scholarship program is certainly a great and valuable experience. I was extremely fascinated by the real mathematics research and its depth, and to see the nice combination of probability theory and geometric group theory. I’ve learned a lot and enjoyed the challenge of research. I also felt grateful that I made friends with other students who are passionate about mathematics as well. I highly recommend vacation scholarship program to fellow students.

And the last, but certainly not the least, a big thank you to my supervisor Dr. Lawrence Reeves who led me onto the beautiful road of mathematics research. Dr. Reeves not only demonstrated himself as a dedicated and talented mathematician, but also a patient, caring and considerate supervisor.

Now I’m more confident that I can live and breathe mathematics for the rest of my life.