Mathematics and Statistics Research Competition 2020

Question 3 – Presentation

Solution:

Given rules

Rules numbers	
1.	$yx = xyc^{k1}$
2.	$zx = xyzc^{k2}$
3.	$zy = x^{-1}zc^{k3}$
4.	$xc = cx$
5.	$yc = cy$
6.	$zc = cz$
7.	$Z^6 = c^{K4}$
8.	$x^{-1}x = xx^{-1} = 1$

From the above given rule 4,

$xc^2 = xc*c$

$= cx*c$

$= c\ (xc)$

$= c\ (cx)$

$= ccx$

$= c^2x$

So, at the same way, we can have

$xc^{ki} = c^{ki}x$ (i = 1, 2, 3, 4)

Similarly, from the given rule 5 and rule 6, we can have

$yc^{ki} = c^{ki}y$ (i = 1, 2, 3, 4)

$zc^{ki} = c^{ki}z$ (i = 1, 2, 3, 4)

Therefore, we can have

Rule 9: $xc^{ki} = c^{ki}x$; $yc^{ki} = c^{ki}y$; $zc^{ki} = c^{ki}z$; (i = 1, 2, 3, 4)

From the given equation $1 = (z^3x)^2$, we can derivate as following:

Derivation	Remark
$1 = (z^3x)^2$	
$=(z^3x)(z^3x)$	
$= z^2 (zx) z^3x$	
$= z^2 (xyzc^{k2}) z^3x$	As the given $zx = xyzc^{k2}$
$= z (zx)yzz^3x\, c^{k2}$	As the rule 9
$= z(xyzc^{k2})\, yz^4xc^{k2}$	$zx = xyzc^{k2}$
$= zxy(zy)\, z^4xc^{k2}c^{k2}$	As the rule 9
$= zxyx^{-1}zc^{k3}z^4xc^{k2}c^{k2}$	$zy = x^{-1}zc^{k3}$
$= zxy\,x^{-1}\,zz^4x\, c^{k2}\, c^{k2}c^{k3}$	As the Rule 9
$= zxyx^{-1}z^5xc^{(2k2+ k3)}$	$c^{k2}\, c^{k2}c^{k3} = c^{(2k2+ k3)}$
$= z\, (xy)\, x^{-1}z^5xc^{(2k2+ k3)}$	
$= z\, (yxc^{-k1})\, x^{-1}z^5xc^{(2k2+ k3)}$	As $yx = xyc^{k1}$ .Therefore, $xy = yxc^{-k1}$
$= z\, (yx)\, x^{-1}z^5x\, c^{-k1}c^{(2k2+ k3)}$	As the rule 9
$= zyxx^{-1}z^5xc^{(2k2 + k3 – k1)}$	$c^{-k1}c^{(2k2+ k3)} = c^{(2k2 + k3 – k1)}$
$= (zy)\, z^5xc^{(2k2+ k3- k1)}$	As the given Rule 8: $xx^{-1} = 1$
$= x^{-1}zc^{k3}z^5xc^{(2k2+ k3- k1)}$	$zy = x^{-1}zc^{k3}$

$= x^{-1}zz^5x\, c^{k3}c^{(2k2+k3-k1)}$	As the rule 9
$= x^{-1}zz^5xc^{(2k2+2k3-k1)}$	$c^{k3}c^{(2k2+k3-k1)} = c^{(2k2+2k3-k1)}$
$= x^{-1}z^6xc^{(2k2+2k3-k1)}$	
$= x^{-1}c^{k4}xc^{(2k2+2k3-k1)}$	$z^6 = c^{k4}$
$= x^{-1}x\, c^{k4}c^{(2k2+2k3-k1)}$	As the Rule 9
$= x^{-1}x\, c^{(2k2+2k3+k4-k1)}$	$c^{k4}c^{(2k2+2k3-k1)} = c^{(2k2+2k3+k4-k1)}$
$= c^{(2k2+2k3-k1+k4)}$	$x^{-1}x = 1$

As anything to the power of 0 equals 1, $c^{(2k2+2k3+k4-k1)} = 1$.

Therefore, **2k2 + 2k3 + k4 - k1 = 0**

School – GWSC

Students - Christabel Yue Yi Liu & Cindy Wu