THE ROLE OF CORRELATED ACTIVITY PATTERN IN EPIDEMIC DIFFUSION: MULTITYPE BRANCHING PROCESS APPROACH

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Introduction

The timing of people's contact patterns plays an important role in the transmission of disease or diffusion of information in a network. The transmission of disease between two people occurs only when both of them are active and there are different activity patterns for each. These patterns will affect the spread because the probability of people becoming inactive varies with their own patterns during the infectious period. For example, the diffusion process could differ in two different networks: a network where people become active randomly in each period and another one where people's active state is highly correlated to their previous behaviour. [1] studies diffusion on some small networks where individuals have different activities patterns and concludes that mixing different types can maximize the diffusion. In this poster, we model the diffusion process by using multitype branching process and simple SIR model, concluding that correlated activity patterns can affect the disease extinction probability in a non-monotonic way.

Model

We define the people being active randomly for each period with probability λ as Poisson people since their active state is memoryless: independent for each period, and the people whose active state is highly or perfectly positively correlated with their previous behaviour as sticky people (assuming either always active (with probability λ) or always inactive (with probability $1-\lambda$)). The diffusion follows the simple SIR model and approximates to a branching process. Assume there is one initial infectious person with the rest of the population being susceptible. Infectious people can pass the disease only in T periods and will be removed after T periods.

In the total population, there are α proportion of Poisson people and $1-\alpha$ proportion of sticky people. Assume the number of susceptible people connected to the infectious person follows $Po(\Lambda)$. Among these connected people, the number of Poisson people D_p and sticky people D_s then follow $Po(\alpha\Lambda)$ and $Po((1-\alpha)\Lambda)$. To successfully pass the disease from one to another, two people need to be connected and be active in the same period. We calculate the probability of an individual successfully passing the disease to another person, (e.g. $Pr(p \to p) = (1-(1-\lambda^2)^T)$). $D_{p\to p}$, the number of Poisson people that an infectious Poisson person successfully passes the disease to, follows $Po(\alpha\Lambda(1-(1-\lambda^2)^T))$. Similarly, we can get the distribution of $D_{p\to s}, D_{s\to p}$, and $D_{s\to s}$.

Let $(Z_t^j)_{t,j\geq 1}$ be the number of connected susceptible people given by person j at time t, which are non-negative integer-valued i.i.d random variables with finite mean. A Galton-Watson Branching Process $(X_t)_{t\geq 1}$ with offspring distribution $\zeta(Z_t^j)$ is a Markov Chain with $X_0 = 1$. Given X_{t-1}, $X_t = \sum_{j=1}^{X_{t-1}} Z_t^j$ [2]

In our model, there are two processes so we define $X_{t,i}^j$ where $i = \{p, s\}$. $X_{t,s}^{(j)} = \sum_{j=1}^{X_{t-1,s}} D_{t,s\to s}^j + \sum_{i=1}^{X_{t-1,p}} D_{t,p\to s}^j$.

This branching process can be simply graphed by:

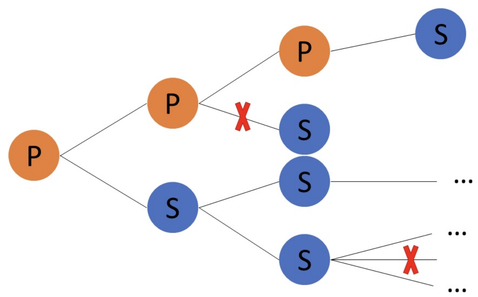

Fig. 1: Branching process with two types: Poisson agents and sticky agents

Black lines denote a connection with two agents. The lines without red cross mean the disease is successfully passed from one to another while the ones with a red cross mean disease fails to be passed because two agents are not active at the same time within the infectious period, though they are connected.

R_0

The mean number of one type of infected people in one generation (also referred to as R_0) can be linked to the previous generation by transition probabilities matrix:

$$\begin{bmatrix} E(X_{p,n}) \\ E(X_{s,n}) \end{bmatrix} = \begin{bmatrix} E(D_{p\to p}) & E(D_{s\to p}) \\ E(D_{p\to s}) & E(D_{s\to s}) \end{bmatrix}^{n+1} \begin{bmatrix} E(X_{p,0}) \\ E(X_{s,0}) \end{bmatrix}.$$

Let **Q** be the transition matrix and r be the Perron-Frobenius root of **Q**.
By [3], if $r > 1$, then $(E(X_{p,n}), E(X_{s,n})) \to \infty$; if $r = 1$, then $(E(X_{p,n}), E(X_{s,n})) = 1$; if $r < 1$, then $(E(X_{p,n}), E(X_{s,n})) \to 0$ and by Markov inequality we know the extinction probability $\to 1$.

Let $\lambda^\star$ be the λ such that r=1, for given α, Λ and T. By calculating the Perron-Frobenius root of **Q**, we conclude that for given Λ and T, increasing the proportion of sticky people (lower the α), can decrease the $\lambda^\star$.

Extinction Probability

The mean number progeny $R_0 < 1$ implies extinction probability $\to$ 1 while $R_0 > 1$ only implies there is a positive chance of non-extinction. The following will investigate more of exact form of extinctino probability.

Let q_i be the extinction probability of disease when $Z_{0,i}^j = 1$ for $i = \{p, s\}$. Similar to [3], for all $i = \{p, s\}$, if $r \leq 1$, then $q_i = 1$; if $r > 1$, then $q_i < 1$. Additionally, let **s** be 2-dimensional nonnegative vector **s** = (s_1,s_2) such that $\|\mathbf{s}\| \leq 1$, the only nonnegative solutions of the equation $\mathbf{f}(\mathbf{s}) = \mathbf{s}$ are 1 and **q**, where $f_i(\mathbf{s})$ is the joint generating function for $(Z_{(T,p)}, Z_{(T,s)})$.

Regrading the joint generating function, we introduce a random variable A (with its realized value a) which represents the number of the active periods of a Poisson infectious person. It is set to capture the dependency of the joint distribution of $D_{p\to p}$ and $D_{p\to s}$. We then solve the equation that:

$$\begin{cases} q_p = f_p(\mathbf{q}) = \sum_{a=1}^{T} e^{[\alpha\Lambda(1-(1-\lambda)^a)(q_p-1)+(1-\alpha)\Lambda\lambda(q_s-1)]} \binom{T}{a} \lambda^a (1-\lambda)^{T-a} + (1-\lambda)^T, \\ q_s = f_s(\mathbf{q}) = e^{[\alpha\Lambda(1-(1-\lambda)^T)(q_p-1)+(1-\alpha)\Lambda\lambda(q_s-1)]}. \end{cases}$$

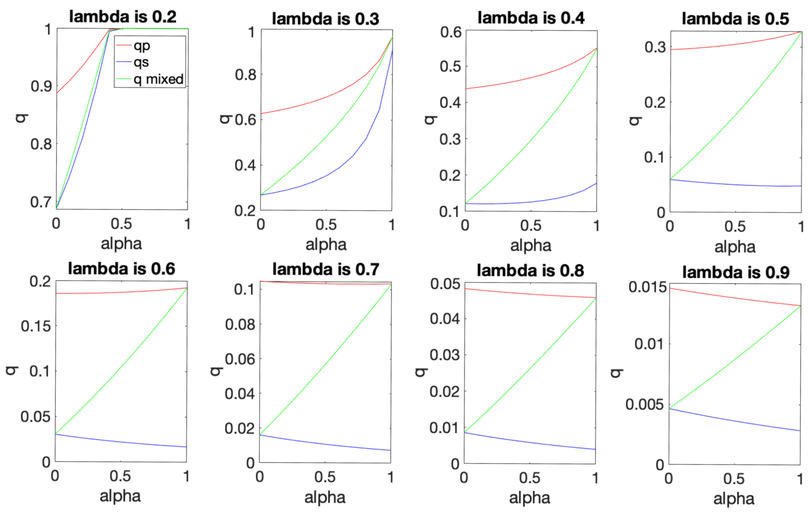

Fig. 2: Extinction probability 1

The diagrams show a decrease in weighted extinction probability when there are more sticky people in the population ($\alpha \downarrow$). This is because the extinction probability of sticky type as the initial person (q_s) is lower than that of Poisson type as the initial person (q_p), which means that the weighted extinction probability is dominated by q_s. This is because that if a person is randomly selected in the total population and we perform the branching process, then more sticky people in the population can improve the diffusion. The intuition is that once sticky people got infected, they are active onward and they can pass the disease successfully once their susceptible connections become active at least once within their infectious period. This makes sticky people better senders.

However, given one type of the initial person, adding more sticky people in the population affects the extinction probability in a non-monotonic way. When λ is small, e.g. 0.2-0.4, both q_p and q_s are lower when increasing the number of sticky people. When λ is relatively large, e.g. >0.5, both q_p and q_s start to change the trend - increase - when increasing the number of sticky people. A possible reason is that the probability of a Poisson/sticky person successfully passing the disease to a Poisson person is higher than that of them passing the disease to a sticky person, and their gap becomes larger when λ is relatively large, even though infectious people are more likely to have more connected susceptible sticky people in both branching processes when α increases.

Extension of other type of sticky person

To model more realistically, we define another type of sticky people, which decreases their 'stickiness': their active status will maintain until infected and they have probability $1-\lambda$ to become inactive in the next period. This extension includes an incubation period in the case of the disease model.

Alternatively, we apply the similar calculation and the result is that proportion of sticky and Poisson people that minimizes the weighted extinction probability varies with λ:

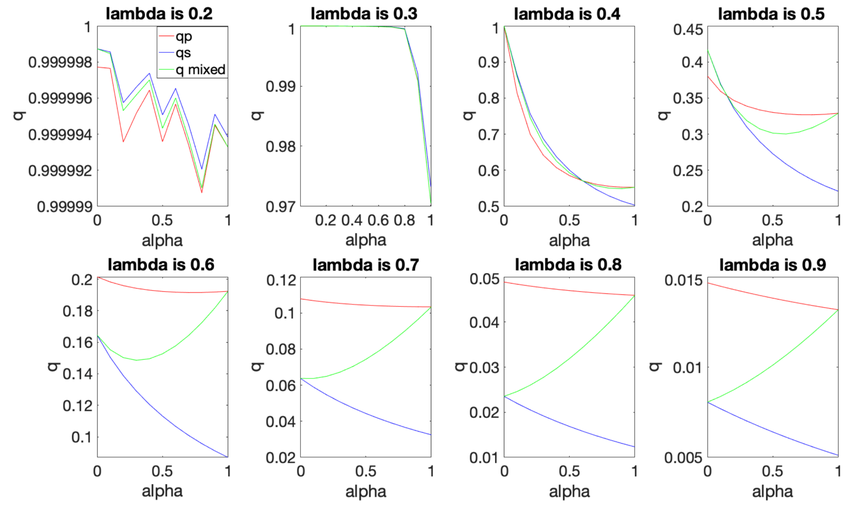

Fig. 3: Extinction probability 2

We see that for small λ, q_s takes a lead role which makes weighted extinction probability higher when increasing the number of sticky people. However, from $\lambda = 0.4$ to 0.5, it demonstrates a clear turning point of weighted extinction probability, indicating that a certain mixed combination of sticky and Poisson people can maximize the diffusion. When λ becomes even larger, extinction probability follows a similar pattern to the previous one as large λ increases the new sticky people's 'stickiness' to approximate to the previous one. Also, both q_s and q_p show an upward trend as α decreases, indicating that 'stickiness' pattern impedes the diffusion process if the first infectious person's type is fixed.

Concluding Remarks

This poster discusses the effect of different activity patterns (sticky or Poisson) on the spread of a diffusion in a network that is well approximated by a branching process. Two cases regarding types of sticky people are considered: No latent period and latent period. We conclude that the effect is determined by the degree of correlation of sticky people and whether the initial infectious person is given.

In the first case, if the initial infectious person is randomly selected based on the proportion of Poisson and sticky people in the total population, perfectly positively correlated activity pattern can maximize the diffusion while if the initial infectious person's type is given (either s or p), sticky people affect the extinction probability in a non-monotonic way. In the second case, heterogeneity in the activity pattern might improve the diffusion if initial infectious person is randomly selected while sticky people impede the diffusion if the initial infectious person's type is given. This result can guide policy implementation for improving the spread of information and transmission of disease based on the different correlated activity patterns in the society.

The next step would be to investigate the robustness of the offspring distribution with distribution other than Poisson. Also, we can consider adding more heterogeneity types to improve the practicability of the model and modelling the diffusion in two structure: household and global one.

Acknowledgements

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References

[1] Mohammad Akbarpour and Matthew O. Jackson. "Diffusion in networks and the virtue of burstiness". In: *Proceedings of the National Academy of Sciences of the United States of America* 115 (July 2018), E6996–E7004.

[2] Krishna B. Athreya and Peter E. Ney. *Branching Processes.* Grundlehren der mathematischen Wissenschaften. Springer-Verlag Berlin Heidelberg, 1972. ISBN: 9783642653711.

[3] Csernica T. "EXTINCTION IN SINGLE AND MULTI-TYPE BRANCHING PROCESSES". In: (2015).