Discrete-time rank-dependent branching processes and fighting processes in the vervet monkey population

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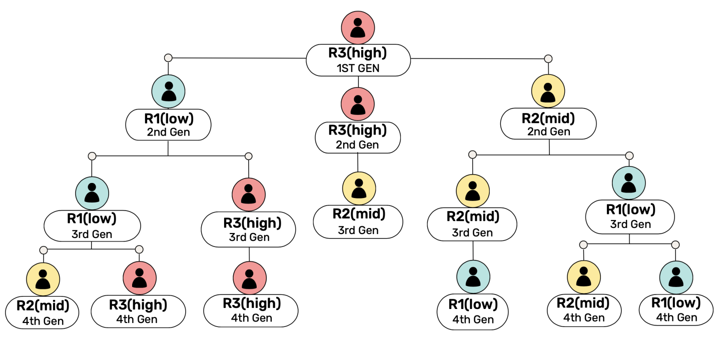
Introduction

The vervet monkey population features a social structure with a dominance hierarchy, which always have some differences between individuals of different ranks. These ranks could affect monkeys' competition for food, territorial distribution, mate choice and survival situation. For each individual, the initial rank is given by their parent(s), and after that, it may depend on their age, physical fitness or social interactions. To simplify the model simulation, the only nurture we consider that affects the ratings is **fighting**.

This poster demonstrates the **birth-fighting-death** processes of **female** vervets for several generations using three-type discrete-time rank-dependent branching processes.

Model Structure

The Family Tree



Premises and Settings

- Time represents ***successive generations*** & each individual only has ***one-generation lifetime***;
- Ratings (mother) > Ratings (daughter1) = Ratings (daughter2) = ... ;
- Rank each female monkey from "***1(low)-2(mid)-3(high)***" via their current ratings;
- Each rank has its own offspring distribution (as shown in the table below, where k : #children);

	$P(k=0)$	$P(k=1)$	$P(k=2)$	$P(k=3)$
R1	0.35	0.3	0.25	0.1
R2	0.25	0.275	0.35	0.125
R3	0.125	0.375	0.375	0.125

- Fighting only happens between ***two*** individuals in the same generation;
- The number of fights: ***"m"*** in a generation is ***population-size-dependently distributed***.

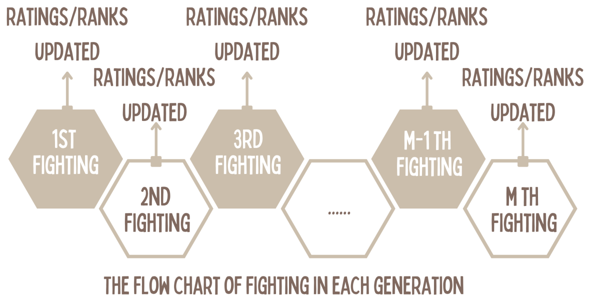


Figure 1. Vervet monkey female with a baby" (Sharp 2009)

Algorithms/Theoretical Basis

1. The *number of fights (m)* in each generation

We can reasonably speculate that the number of fights is positively correlated with the population size. For a generation with size n, we assume that the expected number of fights is equal to n. Then, we can simulate m using a normal distribution with $\mu = n, \sigma = 1$ (Figure 2) and round it to the nearest non-negative integer.

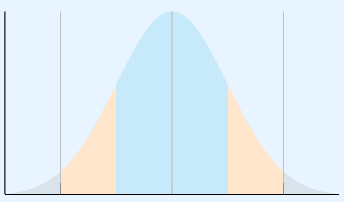
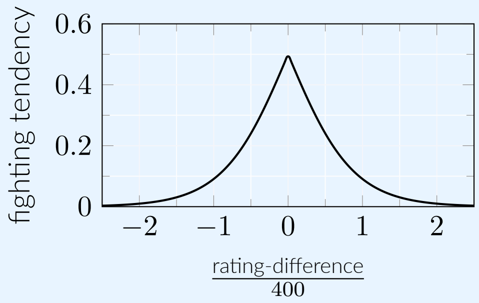


Figure 2. A Normal Distribution with $\mu = n, \sigma = 1$

2. Selection of a *fighting pair*

- Pick out the top 25% monkeys from this generation according to their ratings to form a group (with size k) willing to initiate a fight;
- Select one (player A) to launch the fight: number all members in this group from 1 to k in a rating-ascending order, then the probability of being chosen is determined by their own order divided by $\sum_{i=1}^{k} i$ (higher ratings are considered more aggressive);
- Choose the other one (player B) from the rest of *whole* population to fight with A based on their fighting tendencies, where the fighting tendency of each pair is:

$$\frac{1}{1+10^{|\text{rating-difference}|/400}}$$



3. *Elo rating* system

The Elo rating system, created by *Arpad Elo*, is widely used in measuring the relative skill levels of players in some sports and board games nowadays (*Elo rating system, 2023*).

Mathematical Details: (also in some real games)

$$P_{AB} = P_{(\text{A wins over B})} = \frac{1}{1+10^{(R_B-R_A)/400}}$$
$$P_{BA} = P_{(\text{B wins over A})} = \frac{1}{1+10^{(R_A-R_B)/400}}$$
$$R'_A = R_A + K(S_{(\text{the actual score of A})} - P_{AB})$$
$$R'_B = R_B + K(S_{(\text{the actual score of B})} - P_{BA})$$

where $R_{A(B)}$: the current rating for $A(B)$; $R'_{A(B)}$: a new rating (after fighting) for $A(B)$;

$S_{(\text{the actual score})} = \begin{cases} 1 & \text{if wins} \\ 0 & \text{if loses} \end{cases}$; K: K-*factor*, the maximum possible adjustment per fighting. For here, we choose $K = 32$, which is relatively high (usually for ratings below 2100, so that the system would be more sensitive to recent fights).

Simulation Results

The fighting processes in a fixed-size generation

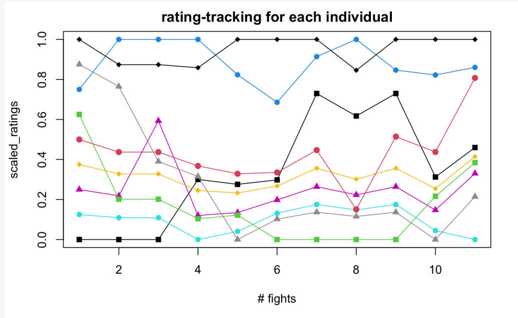


Figure 3. Generation with initial ratings: (125, 130, 135, 140, 145, 150, 155, 160, 165), relatively close

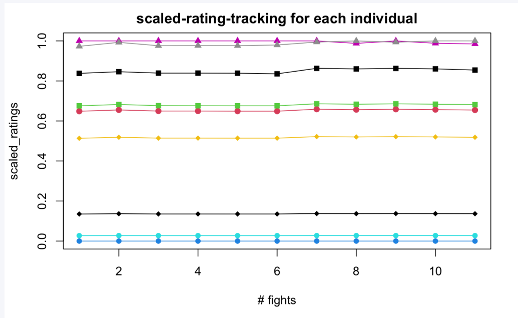


Figure 4. Generation with initial ratings: (1800, 300, 2100, 1200, 1450, 1500, 250, 2050, 500), relatively far apart

- Pairs with (relatively) close initial ratings are more affected than those with larger rating-differences.
- High scores drop more than low scores when they lose and increase less when they win.
- More fights happen between high-rated pairs.

The Evolution of (scaled) Score Distribution

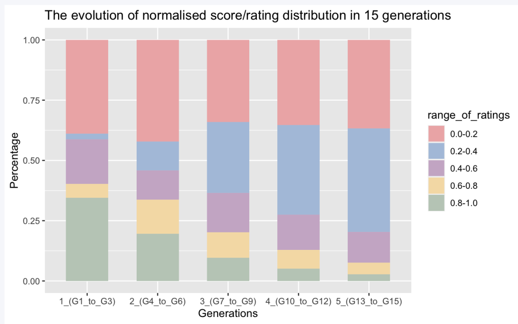


Figure 5. The evolution starting with (1250, 1500, 1750)

For different types of initial population, the evolutions of re-normalised score distribution are quite similar: the size of both extreme scores gradually decreases, and mid-ratings are becoming dominant hierarchies (especially for scaled ratings in (0.2-0.4)).

Further Extensions

- Further research on realistic ratings that have practical significance.
- Real offspring distribution should be tested.
- Explore situations where some monkeys may live longer and death can be an outcome of a fight.
- Explore the practical inheritance of ratings.

References

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