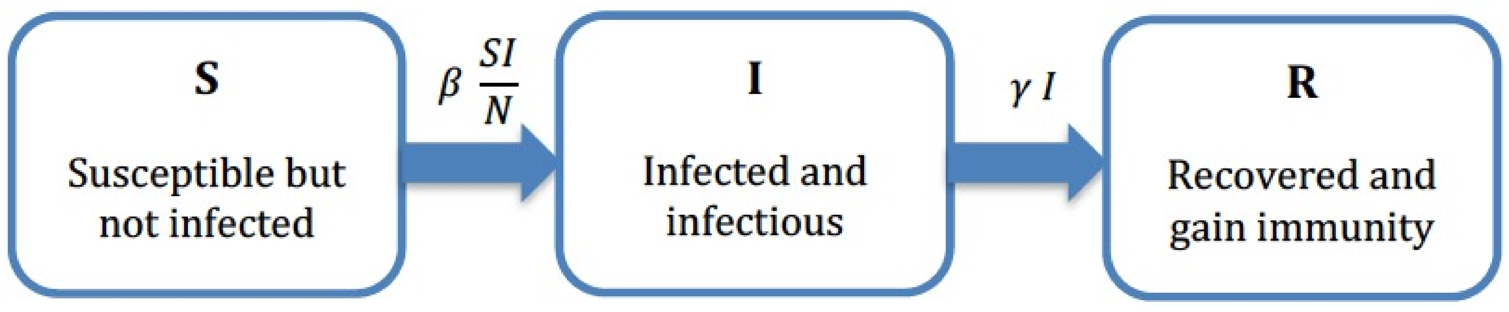
Mathematical Modelling of Infectious Disease Epidemic

Ruoxin Liu, Supervised by Dr. Anthony Morphett

The University of Melbourne

Introduction

Infectious disease epidemic can be described using the SIR Model.



Schematic Diagram

$\beta =$ *transmission rate* ; $\gamma =$ *recovery rate*

This project involves exploring the suitability of the SIR Model when describing the progression of a simulated epidemic that spreads through shaking hands. In particular, a continuous version of SIR model will be examined regarding its performance as population size increases. This project might be adapted for educational purposes at high schools or universities.

The SIR Model

In the following context, define:

$s = \frac{S}{N}$, $i = \frac{I}{N}$, and $r = \frac{R}{N}$, where N is the population size.

Differential equations for the continuous version of SIR Model:

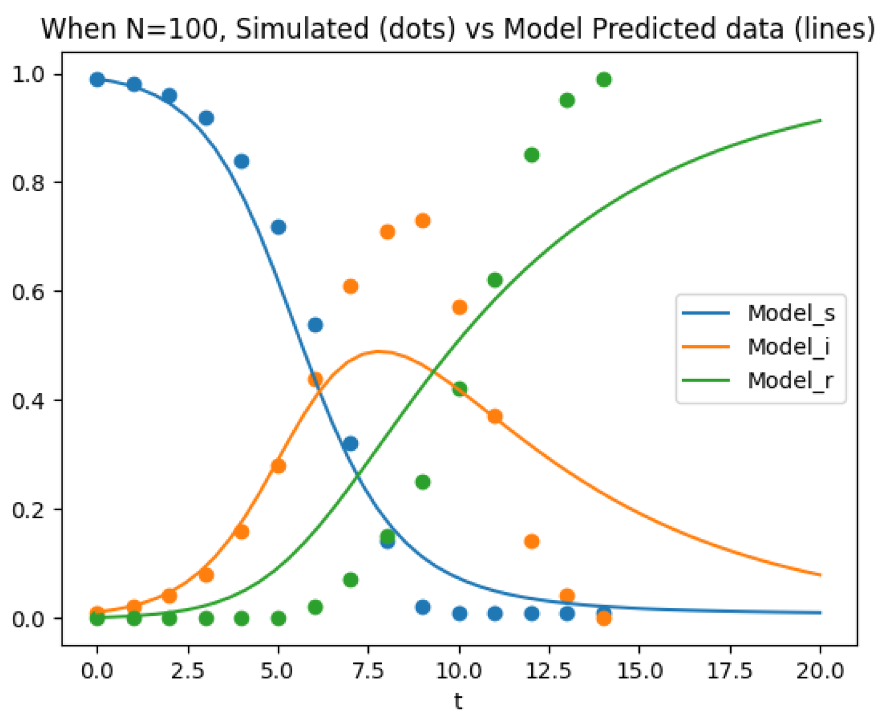
$$\frac{ds}{dt} = -\beta si \qquad \frac{di}{dt} = \beta si - \gamma i \qquad \frac{dr}{dt} = \gamma i$$

Assumptions in the model here:

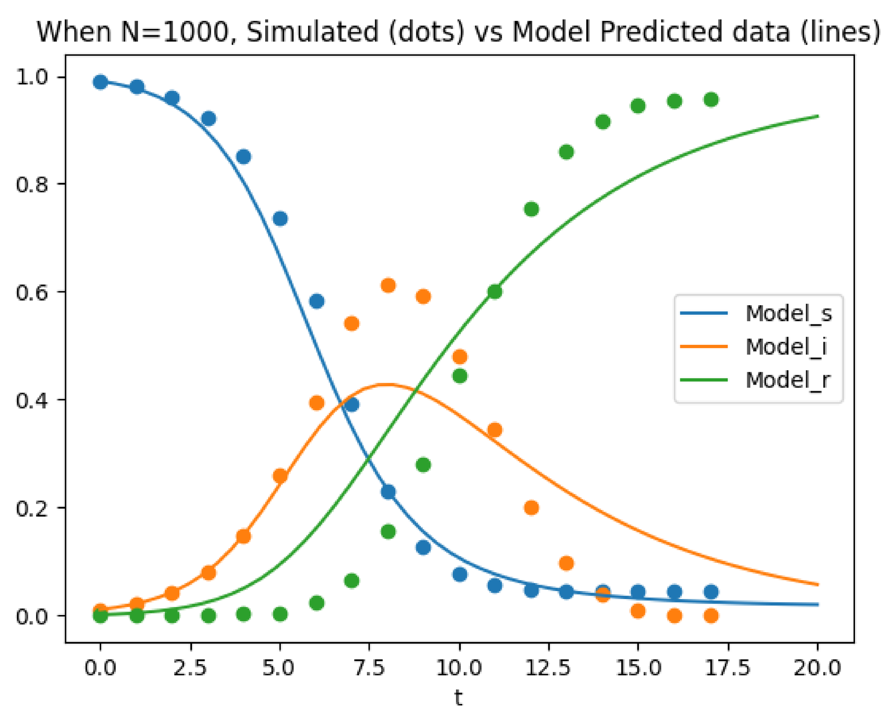
- N is constant in each case
- Every individual shakes hands with 10 people
- Every infectious person requires 5 periods to recover
- Initial settings: $s_0 = 0.99$, $i_0 = 0.01$, $r_0 = 0$

SIR Model Fitting for different population sizes: When Parameters Are Fixed

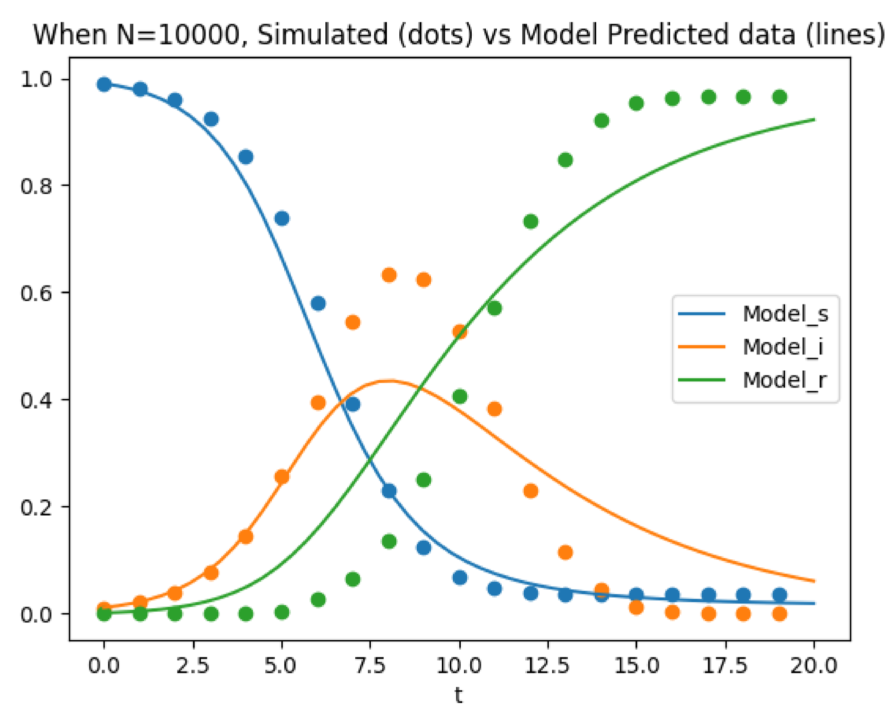
Fit the SIR Model and obtain the values for beta and gamma that minimize the Residual Sum of Squares (RSS). This process was repeated 10 times for population size N=100, 1000 and 10000 respectively. The plots below are typical one selected for display purposes.



$\beta = 0.980; \gamma = 0.191$



$\beta = 0.966; \gamma = 0.227$



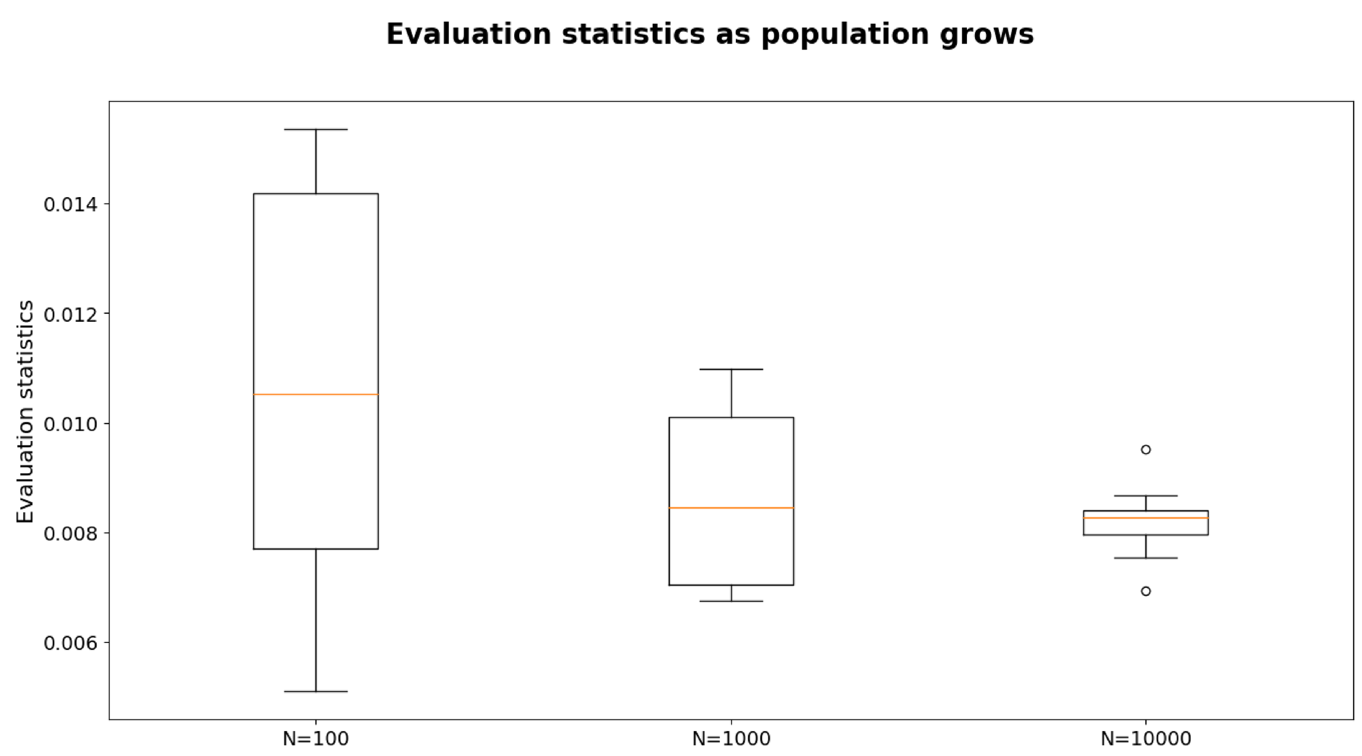
$\beta = 0.962; \gamma = 0.221$

While the model predicts the dynamics of susceptible well to some extent, the infectious and recovered parts are not satisfactory. Comparing the model and simulated data, SIR Model does not qualitatively give a better fit when N increases, because the peaks of 'i' are underestimated and the dynamics of 'r' by the model does not match observed data when t gets larger.

Quantitative Analysis of Results

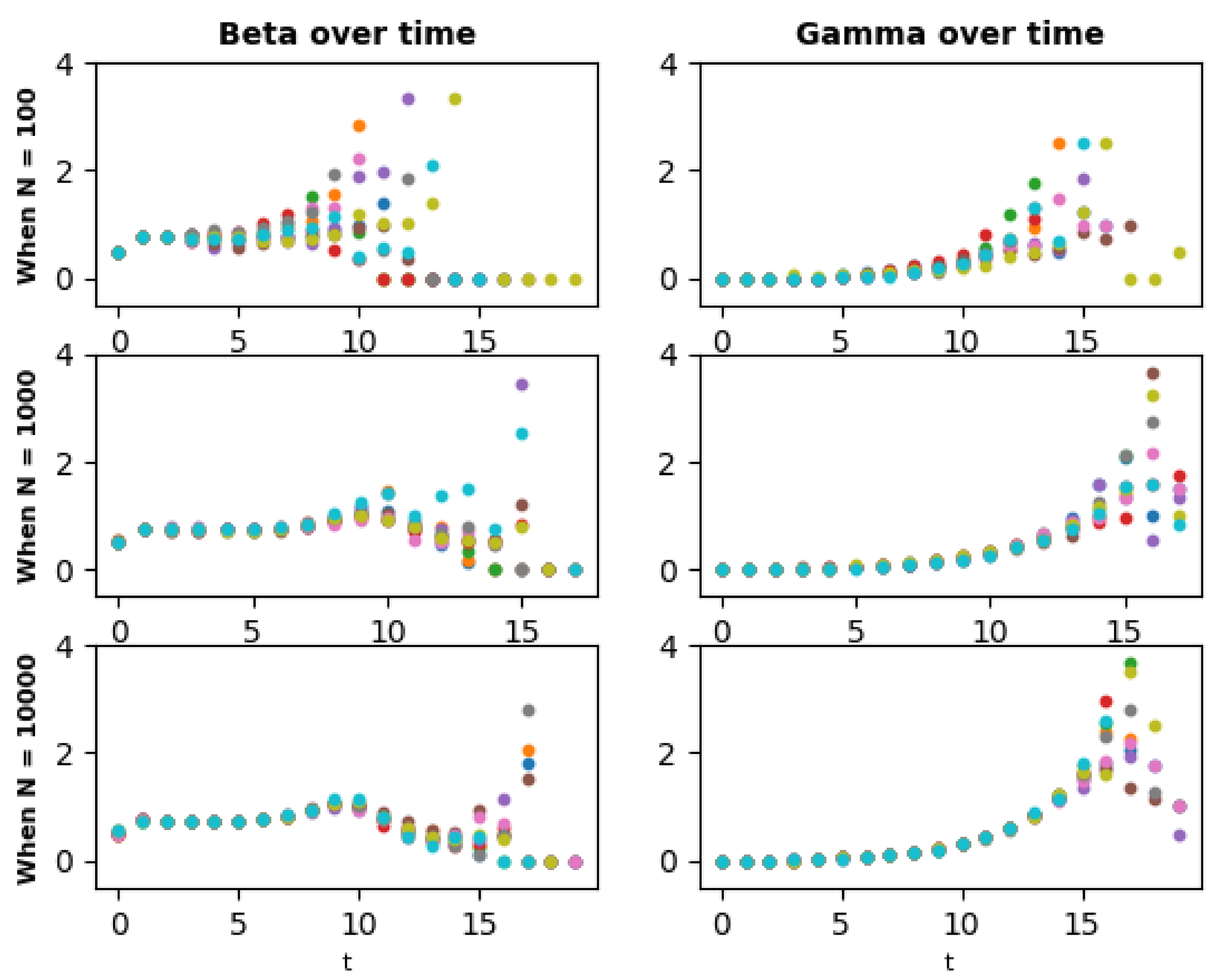
Define Evaluation statistics: $RSS/no.points$.

Regression analysis between the evaluation statistics and N does not apply here due to unequal variance among different sets of data. We also observe that the evaluation statistics tend to be less variable when population size is larger.



Conclusion: the continuous version of SIR Model does not give a better fit as population size increases.

Patterns for Beta and Gamma over time



Outliers have been removed

In contrast with the discrete version of SIR Model, beta and gamma values could not be calculated directly in the continuous version. By approximating the gradients of s, i, r curves respectively at t = 0, 1, 2, ..., parameter values over time are computed and shown in the plots above for different classes of populations.

Discussion

- In this program, I also explored patterns for beta and gamma over time using the discrete model in Excel. Comparing the results, it turns out that the trend for beta in the discrete model coincides with the continuous one when t is larger than 10; as for gamma, two versions of models behave similarly when t is smaller than 10. This indicates that the choice between continuous or discrete SIR Model may affect the modelling result if we further fit the SIR model with beta and gamma values that changes with t.
- In addition, we also modified the assumption of H = 10 to $H = N/10$. It does not turn out to give a better fit either. The results are not attached here due to the word limit.

Acknowledgements

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