Supercharging Charging: A Decentralised Coordination Model of Plug-in Electric Vehicle Charging with Vehicle-to-Grid Capabilities

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INTRODUCTION

The infrastructural and logistical issues with facilitating charging for the recent sudden eruption of plug-in-electric vehicle (PEV) adoption remain a key challenge for policymakers around the world. Individual PEV owner charge their vehicles according to what is locally optimal for them and as a result, we observe concentrated charging activities during the evening hours where non-PEV demand is also at its peak. This uncoordinated behavior causes overloading stress on the grid and incurs unnecessarily high costs for their owners.

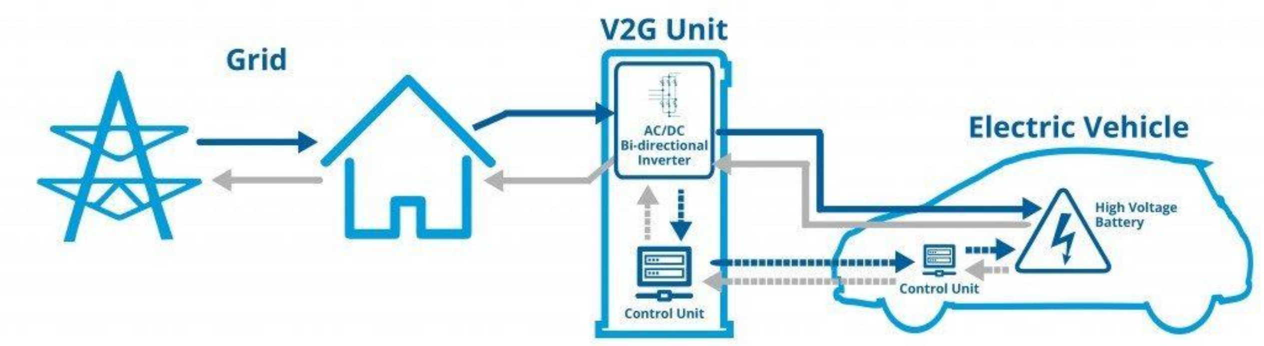

Figure 1: V2G Explainer Diagram
Credits: ZHAW School of Engineering

However, the introduction of vehicle-to-grid (V2G) charging technology, which allows PEVs to feed excess or idle power back into the grid to alleviate load stress, opens up a new horizon for how PEV charging can be coordinated and optimised.

The focus of my vacation project is extending the aggregative game model of PEV charging agents introduced by Belgioioso & Grammatico (2003) through the addition of V2G capabilities to each agent. My V2G-inclusive model is solved using the Preconditioned Forward-Backward (pFB) algorithm in Python.

PROBLEM FORMULATION

Let $x_i \in R^n$ represent the charging decisions of each agent $i \in \mathrm{I} \coloneqq \{1,2,3,\dots N\}$ across the time horizon of a full day, where $n = 24$ hours. Each agent is subject to their decision set Ω_i, determined by **maximum per-period charge** $\bar{x}_i$, **minimum total daily charge** l_i and **final desired charge state** η_i. For simplicity, we denote cumulative charging decisions of all agents in a particular time period t as $\boldsymbol{x} \coloneqq col(x_1, x_2, x_{3,\dots,} x_N)$ and cumulative charging decisions of all agents except agent i as $\boldsymbol{x}_{-\boldsymbol{i}} \coloneqq col(x_1, x_2, x_{i-1}, x_{i+1,\dots,} x_N)$.

Thus, the resultant system of optimisation problems is modelled as such for $\forall i \in I$:

$$\begin{cases} \min\limits_{x_i \in R^n} \quad J_i(x_i, \boldsymbol{x}_{-\boldsymbol{i}}, u_i) \coloneqq g_i(x_i) + p(\frac{1}{N}\sum_{i=1}^{N} x_i)^T x_i - q_i u_i & \textbf{(Cost Function)} \\ s.t. \quad x_i \in \Omega_i & \textbf{(Local Decision Set)} \\ \quad \sum_{i=1}^{N} x_i(t) \le NK(t), \forall t = 1,2,3,\dots,n & \textbf{(Coupling Constraint)} \end{cases}$$

Cost Function

- $g_i(x_i)$ models **unique individual costs** like battery degradation and time preferences etc.
- $p(\frac{1}{N}\sum_{i=1}^{N} x_i)^T x_i$ models **charging cost** based on aggregate decisions and non-PEV demand
- $q_i u_i$ models **cost-offsetting V2G income** with amount u_i modified by price/efficiency level q_i

Local Decision Set

$x_i \in \Omega_i$:

$$\begin{cases} 0 \le x_i \le \bar{x}_i & \text{(a)} \\ \sum_{t=1}^{n} x_i(t) \ge l_i & \text{(b)} \\ 0 \le u_i \le \eta_i & \text{(c)} \end{cases}$$

Therefore, each agent $i \in \mathrm{I} \coloneqq \{1,2,3,\dots N\}$

a) can never exceed its maximum charge $\bar{x}_i$
b) must charge at least l_i over the course of the day
c) cannot sell more than its final charge state η_i at the end of the day

Coupling Constraint

- This models the maximum aggregate charge that the grid can deliver to the electric vehicles in any given period, represented by $K(t)$

ALGORITHM AND DATA

Algorithm 1
Preconditioned Forward-Backward (pFB)

Initialisation: $\delta > \frac{1}{2\gamma}; \forall i \in \mathcal{I},\ x_i^0 \in \mathbb{R}^n$,
$0 < \alpha_i \le (||I_n|| + \delta)^{-1}; \lambda^0 \in \mathbb{R}^n_{\ge 0}$
$0 < \beta \le (\frac{1}{N}\sum_{i=1}^{N} ||I_n|| + \frac{1}{N}\delta)^{-1}$
Iterate until convergence:
1. Local: Strategy update, for all $i \in \mathcal{I}$:
$y_i^k = x_i^k - \alpha_i[\nabla_{x_i} p(\operatorname{avg}(\boldsymbol{x}))^T x_i + \lambda^k]$
$x_i^{k+1} = \operatorname{prox}_{\alpha_i g_i + \iota_{\Omega_i}}(y_i^k)$
$d_i^{k+1} = 2x_i^{k+1} - x_i^k - K$

2. Central coordinator: dual variable update
$\lambda^{k+1} = \operatorname{proj}_{\mathbb{R}^n_{\ge 0}}(\lambda^k + \beta \operatorname{avg}(\boldsymbol{d}^{k+1}))$

Figure 2: pFB algorithm
Credits: Beigioioso & Grammatico (2009)

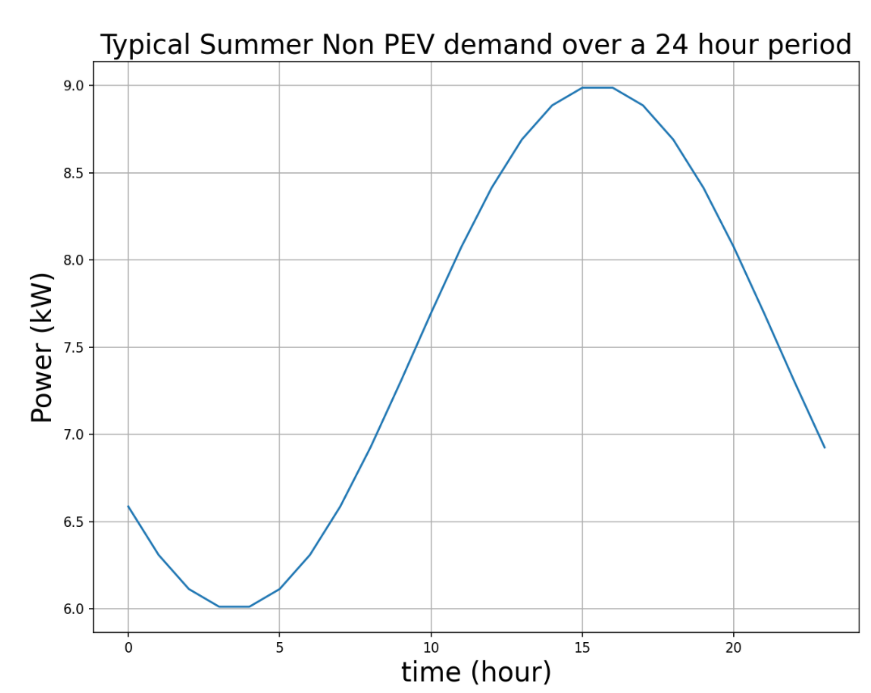

Figure 3: Model **Summer** Non-PEV grid load curve with exaggerated characteristics as a sine function with a singular peak at 3pm.

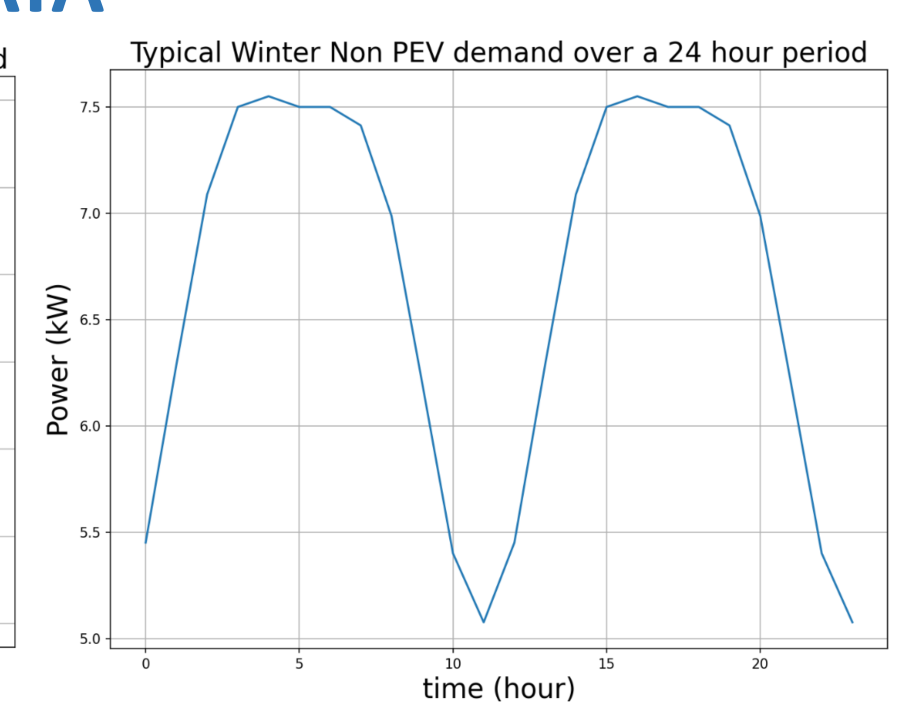

Figure 4: Model **Winter** Non-PEV grid load curve with exaggerated features as a product of sine and cosine functions with dual peaks at 9am and 3pm

RESULTS

Figure 5: Initial random decisions of 10 agents, each colour represents the charging decisions of a particular agent throughout the day. The V2G decisions at the end of the day is highlighted in yellow.

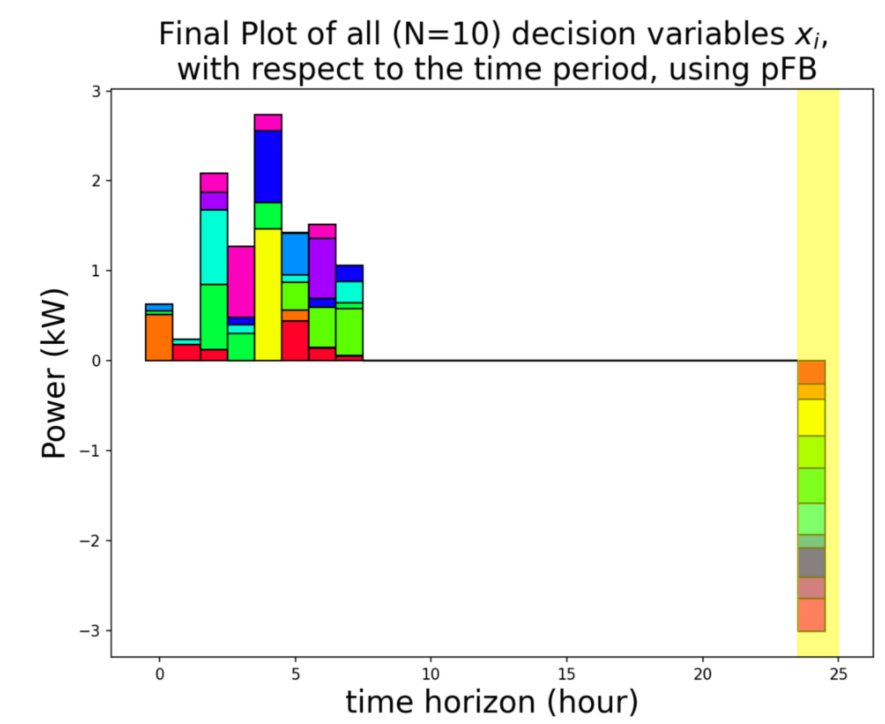

Figure 6: Final converged decisions against a typical **summer** non-PEV demand. Note how it is inverse to the shape of Fig 3 to alleviate grid load at peak times. Convergence is considered achieved when $\varepsilon \le 10^{-3}$.

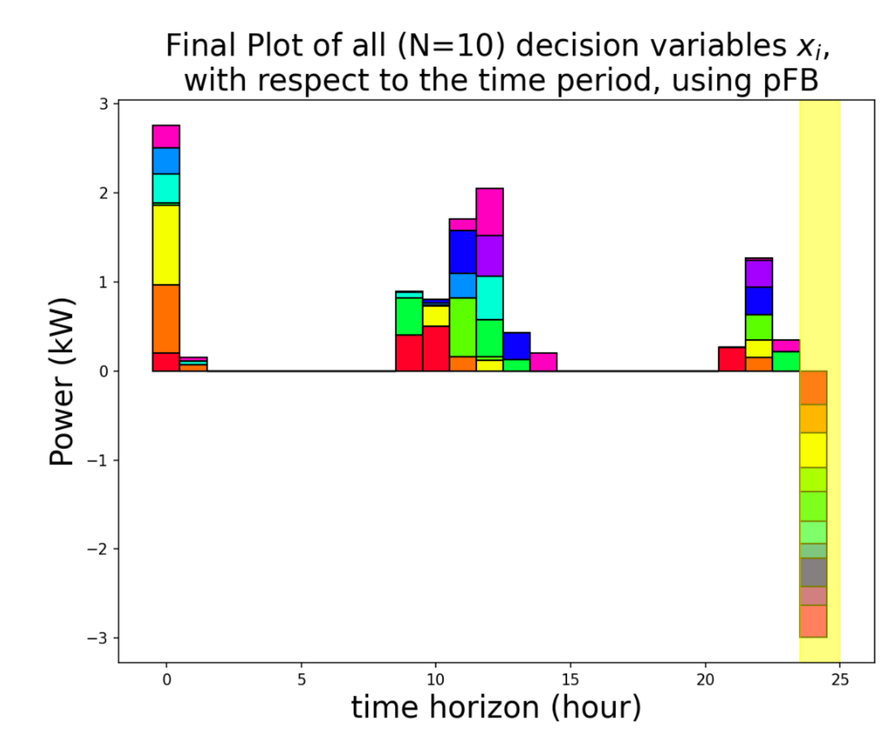

Figure 7: Final converged decisions against a typical **winter** non-PEV demand. Note how it differs from Fig 7 due to their different non-PEV load curves. In contrast, V2G decisions are not impacted significantly.

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