Bayesian Audits for 2-Candidate Elections with Invalid Votes

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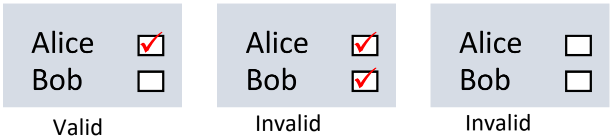
We investigate Bayesian approaches to auditing 2-candidate plurality elections with *invalid votes.* Alice is the reported winner.



The General Bayesian Approach

H_0: Alice did not actually win e.g. $\theta \leq 0.5$

H_1: Alice actually won e.g. $\theta > 0.5$

Compute the *upset probability* $U_n = \Pr(H_0|\text{Data})$ and compare to some threshold ν (e.g. 0.05):

- $U_n < \nu \Rightarrow$ reject H_0; certify Alice as the winner.
- $U_n \geq \nu$ and
  - $n < m \Rightarrow$ increase the sample size and repeat.
  - $n = m \Rightarrow$ do a full recount.

Model (A) – Explicitly modelling invalid votes

- θ: Alice's vote share out of the valid votes
  - Uniform prior, $H_1: \theta > 0.5$
- (n_A, n_B, n_I): In-sample vote counts (Alice, Bob, Invalid)

(Beta) Posterior:

$$f(\theta|\text{Data}) \propto \theta^{n_A}(1-\theta)^{n_B}$$

Comparison with no invalid votes case [1]

Same posterior as (A) but:

- θ: Alice's vote share, $n_B = n - n_A$

Model (B) – Augmented data for invalid votes

Set value ½ to invalid votes and apply to no invalid votes case

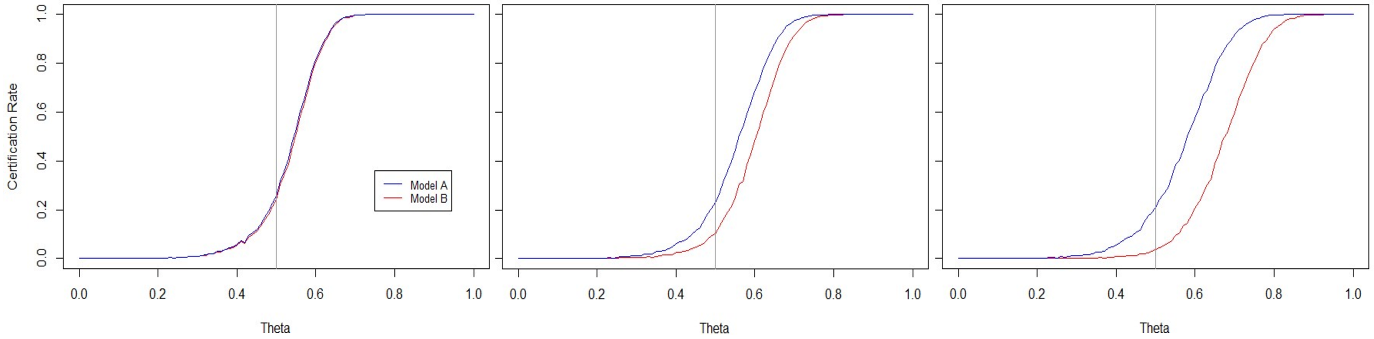
$$g(\theta|\text{Data}) \propto \theta^{n_A+\frac{1}{2}n_I}(1-\theta)^{n_B+\frac{1}{2}n_I}$$

Based on SHANGRLA [2].

Comparison of (A) and (B) via simulation

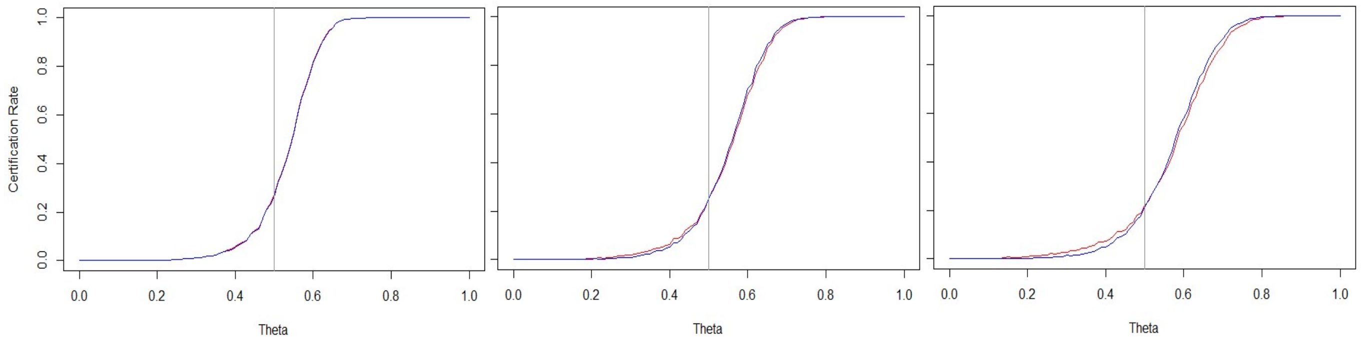
Power $= \Pr(\text{Certification}|H_1 \text{ true})$, *Risk* $= \Pr(\text{Certification}|H_0 \text{ true})$ i.e. miscertification

- $m = 100, \nu = 0.05$, Left to right: $p_I = 0.05, 0.4, 0.6$



As p_I increases, Model (B) is more conservative (lower power, lower risk) than Model (A) for fixed ν.

Recalibration: Increase ν in Model (B) so that worst-case risk (A) = worst-case risk (B).



The models are very similar once we recalibrate.

Acknowledgements

Thank you Damjan for the stellar supervision!

References

[1] Huang, Z., Rivest, R. L., Stark, P. B., Teague, V. J., & Vukcevic, D. (2020, October). A unified evaluation of two-candidate ballot-polling election auditing methods. In International Joint Conference on Electronic Voting (pp. 112-128). Springer, Cham.

[2] Stark, P. B. (2020, February). Sets of half-average nulls generate risk-limiting audits: SHANGRLA. In International Conference on Financial Cryptography and Data Security (pp. 319-336). Springer, Cham`