

The encounter rate of microbes in nutrient rich water

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Introduction

- ▶ Bacteria can transfer genes when they collide.
- ▶ Modelling the encounter rate between microbes teaches us how fast these genes spread.
- ▶ Bacteria undergo Brownian motion [1].
- ▶ Theoretical encounter rate [2]:

$$E = 16\pi DaC^2,$$

where a is the radius, C is the concentration, and D is the diffusivity of the bacteria.

- ▶ Microbes can navigate towards nutrients.
- ▶ This means the previous rate does not hold, because the concentration is not constant.

Simulations

- ▶ We simulated the motion of particles in three dimensions.
- ▶ At each time step, each particle's position is perturbed by $\delta\mathbf{x}$:

$$\delta\mathbf{x} = \sqrt{2D\Delta t} \begin{pmatrix} \pm 1 \\ \pm 1 \\ \pm 1 \end{pmatrix} - k\nabla(P)\Delta t.$$

- ▶ D is diffusivity, Δt is the time step, P is a function modelling the nutrient concentration, k models the attraction to the nutrients.
- ▶ At each time step we can count the number of collisions.

Figure 1

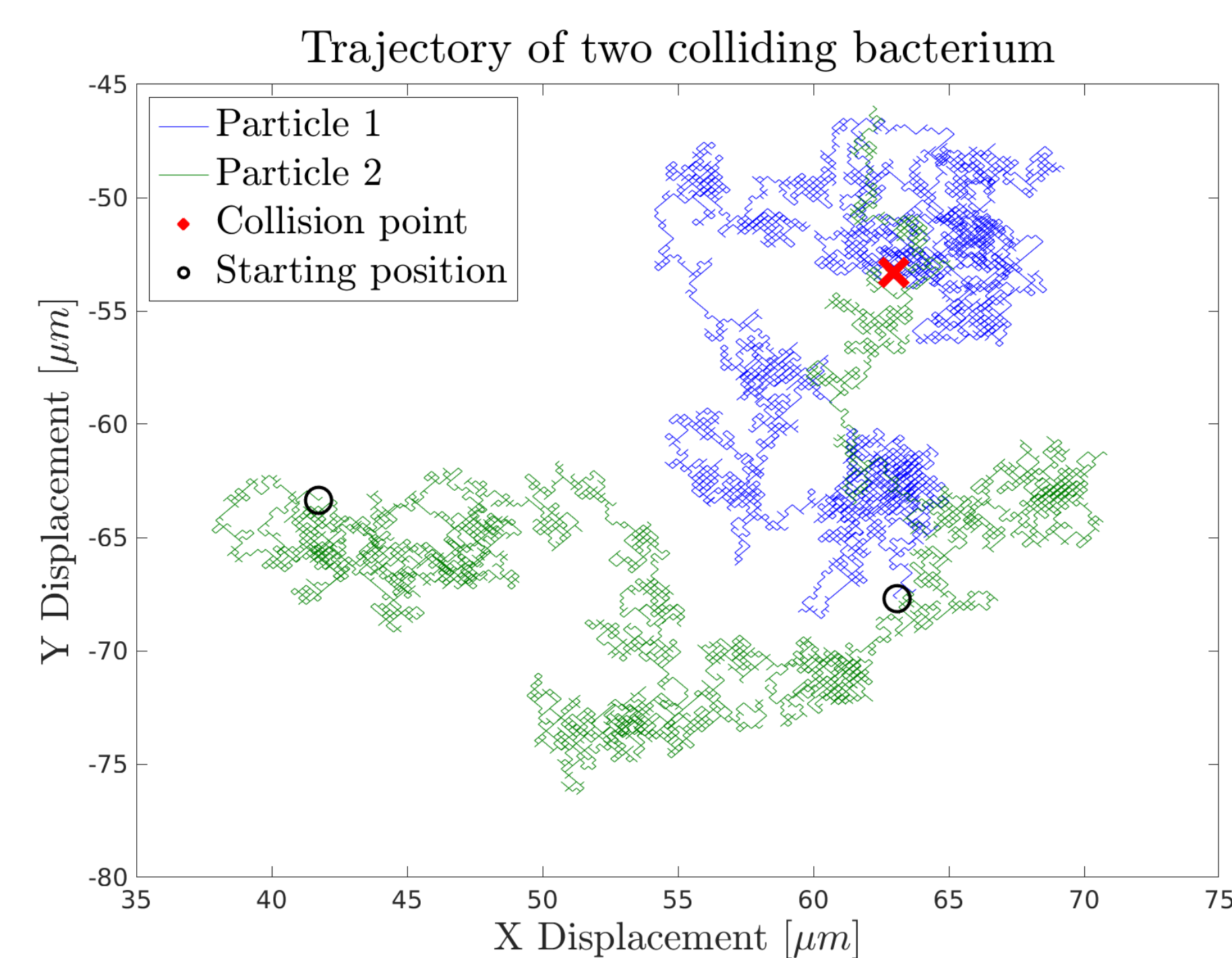


Fig. 1: Simulated trajectories of two colliding particles.

Discrete Modelling

- ▶ We want to know if the overall collision rate follows

$$E = \int_{\Omega} 16\pi DaC(\mathbf{x})^2 d\Omega, \quad (1)$$

where Ω is our domain.

- ▶ We can estimate the concentration using `mvksdensity`, then integrate.

Continuous Modelling

- ▶ The distribution of our particles is modelled by the steady state Convection–Diffusion equation:

$$\nabla \cdot (D\nabla C) - \nabla \cdot (\mathbf{v}C) = 0,$$

where $\mathbf{v}(\mathbf{x})$ is the bias velocity.

- ▶ We find that the solution is

$$C = A \exp\left(-\frac{k}{D}P\right),$$

where A is a constant which depends on the number of bacteria.

Figure 2

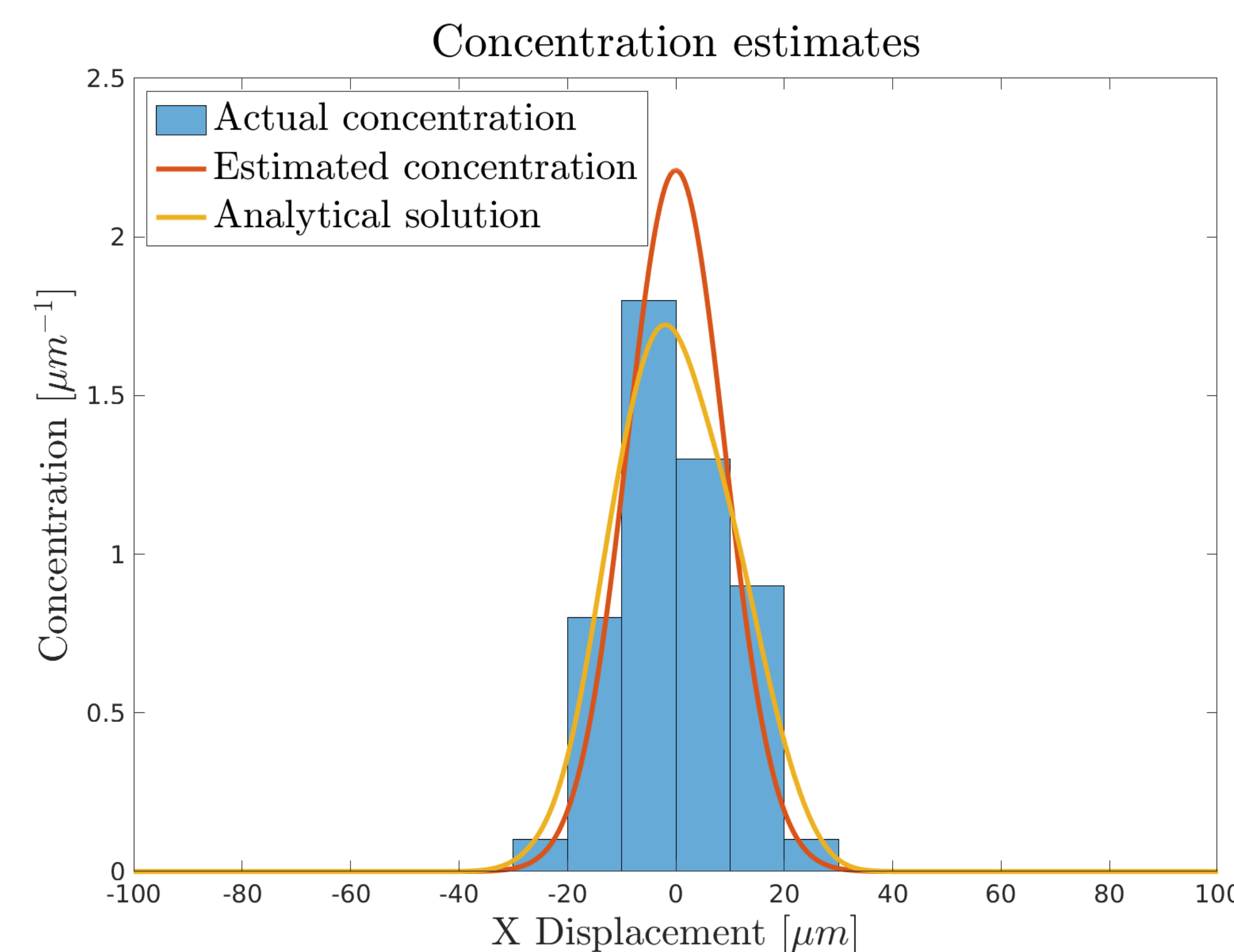


Fig. 2: Comparison of concentration estimate with solution.

Figure 3

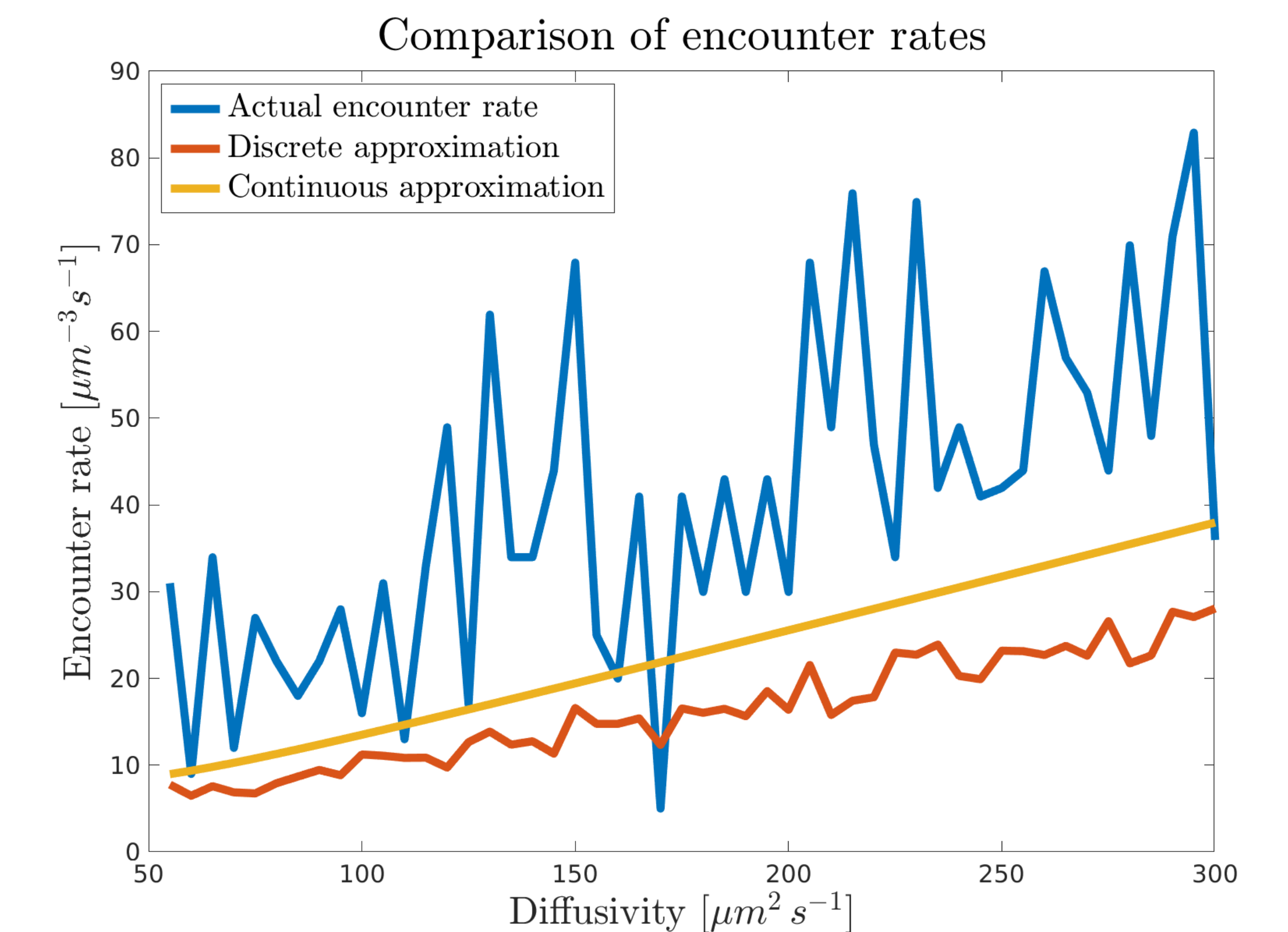


Fig. 3: Comparison of approximate collision rates.

Discussion

- ▶ We want all the ways of estimating the encounter rate to have the same trend.
- ▶ We can compare the estimates over a range of diffusivities and radii.
- ▶ As shown in Figure 3, they have the same trend.
- ▶ We can conclude that (1) provides a useful estimate for collision rate in nutrient rich water.

Further Research

- ▶ This model could be used in a variety of scenarios, where we have a non-uniform concentration profile.
- ▶ Many physical phenomena could be investigated, modelled and compared with the work presented here, such as:
 - ▶ Spread of infection,
 - ▶ Information spread.

References

- [1] H. C. Berg, *Random walks in biology*. Princeton, N.J: Princeton University Press.
- [2] K. Moor, M. Diard, M. E. Sellin, B. Felmy, S. Y. Wotzka, A. Toska, E. Bakkeren, M. Arnoldini, F. Bansept, A. D. Co, T. Völler, A. Minola, B. Fernandez-Rodriguez, G. Agatic, S. Barbieri, L. Piccoli, C. Casiraghi, D. Corti, A. Lanzavecchia, R. R. Regoes, C. Loverdo, R. Stocker, D. R. Brumley, W.-D. Hardt, and E. Slack, "High-avidity IgA protects the intestine by enchainning growing bacteria," *Nature*, vol. 544, no. 7651, pp. 498–502, Apr. 2017. [Online]. Available: <https://doi.org/10.1038/nature22058>