Self-organised criticality in real world systems

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Introduction

The concept of **Self-organised criticality (SOC)** was first introduced by Per Bak, Chao Tang, and Kurt Wiesenfeld in 1987 [1]. SOC refers to a phenomenon in complex systems where interacting components collaborate to bring the system to a critical state without any tuning of the control parameters. In this critical state, any minor perturbation can lead to a major disturbance. This concept has been applied across fields as diverse as ecology, mathematics, economics, neuroscience and others. For example, tectonic plates move constantly but when pressure builds up and reaches a critical state, an earthquake will occur.

The Bak-Tang-Wiesenfeld (BTW) model (or "Sandpile model") can be used to model SOC. The frequency distribution of the size of disturbances has been found to follow the power law distribution $f(x) \propto x^{-\alpha}$ (positive constant α) for complex systems. We will use a 2-dimensional sandpile model to analyse this property, and then investigate self-organised criticality in the brain by using a modified version of the sandpile model to simulate neuronal activities.

BTW Sandpile Model

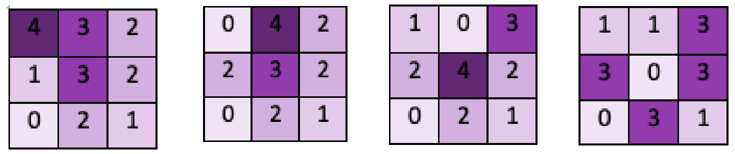
Consider a lattice of size N × M. We drop grains of sand randomly onto cells of this rectangular lattice, one at each time step (t). Let $h(i, j)$ be the number of sand grains present at site (i, j) at time t where $i \in [1,2,\ldots,N]$, $j \in [1,2,\ldots,M]$. Whenever a site exceeds the threshold value (4 sand grains), a topple occurs, distributing all the four sand grains to its four neighbours such that

$$h(i, j) = h(i, j) - 4$$

$$h(i \pm 1, j) = h(i \pm 1, j) + 1$$

$$h(i, j \pm 1) = h(i, j \pm 1) + 1$$

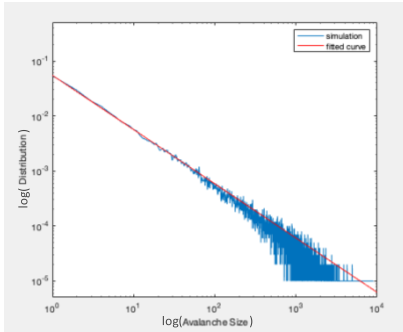
Grains that fall outside the range of the lattice are lost.



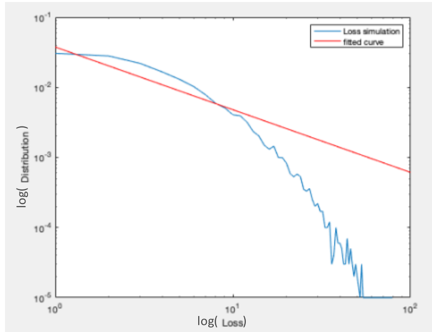
- *Avalanche size* is defined as the total number of topples which occur after adding a single sand grain until the system reaches a stable state.
- *Loss* is defined as the total number of sand grains lost off the lattice after adding a single sand grain until the system reaches a stable state.

Simulation: lattice of size 50 × 50 with 100,000 time steps

We plot the frequency distribution of *Avalanche size* and *Loss* on a log-log scale and fit $y = \beta x^{-\alpha}$ to the curves.



α = 0.9858



α = 0.8942

Modified Sandpile Model

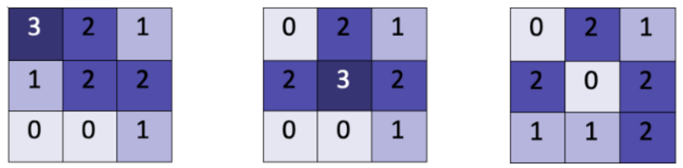
Self-organised criticality in the brain has been reported in many studies. Scale-free avalanches have been observed in neural activities using brain functional neuroimaging (fMRI) [2]. Neurons use electrical signals to communicate with each other. When the voltage reaches a threshold value, the neuron fires an action potential, distributing the signals to its neighbours. It works very much like the sandpile model.

However, the transmission of neural signals is unidirectional (from dendrites to axons). Per Bak constructed a model to investigate how a monkey's brain solves problem [3]. In his model, each neuron is connected with three neurons at the row below, which reflects the unidirectional characteristic. We modify the sandpile model based on Per Bak's model to simulate neuronal activities.

Threshold value = 3 $\quad h(i, j) = h(i, j) - 3$

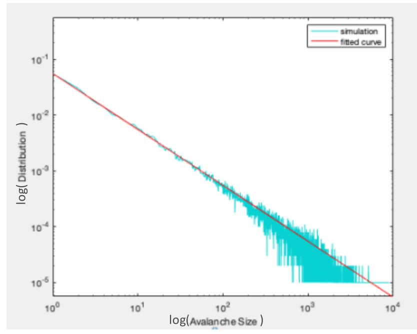
$$h(i + 1, j) = h(i + 1, j) + 1$$

$$h(i + 1, j \pm 1) = h(i + 1, j \pm 1) + 1$$

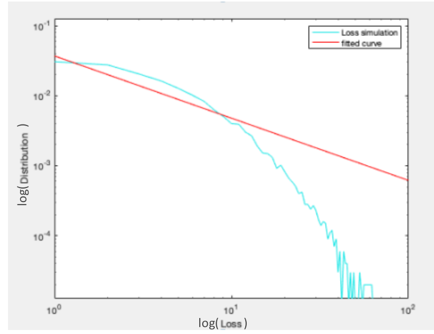


Modified model simulation: lattice of size 50 × 50 with 100,000 time steps

We plot the frequency distribution of *Avalanche size* and *Loss* on a log-log scale and fit $y = \beta x^{-\alpha}$ to the curves.



α = 1.002



α = 0.8885

Conclusion

Our simulation of BTW sandpile model provides clear evidence of a power law distribution, especially for *Avalanche size*. The power law trend for *Loss* is less than that of *Avalanche size* since it could be zero even if there is an avalanche, which makes the graph less precise, but it still follows a power law distribution.

Our simulation of the modified sandpile model also gives a clear indication of the power law distribution. It implies that the neural system in the brain is a self-organised critical system. The α value for *Avalanche size* is slightly higher than that of the BTW model. One possible explanation could be that the unidirectional distribution leads to a lower frequency of large avalanches.

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References

[1] Bak, P., Tang, C. and Wiesenfeld, K., 1987. Self-organized criticality: An explanation of the 1/f noise. *Physical Review Letters*, *59*(4), p.381.
[2] Cocchi, L., Gollo, L.L., Zalesky, A. and Breakspear, M., 2017. Criticality in the brain: A synthesis of neurobiology, models and cognition. *Progress in Neurobiology*, *158*, pp.132-152.
[3] Bak, P., 2013. *How Nature Works: The Science of Self-organized Criticality*. Springer Science & Business Media.