Students who have not completed the Conics option of Maths C also need to revise the following topic.

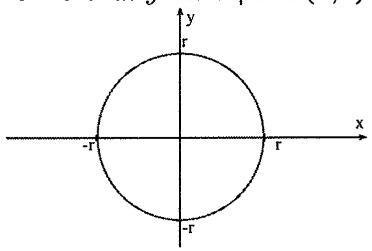
4.1 Graphs of Circles, Ellipses and Hyperbolae [Chapter 11.4]

4.1.1 Graphs of circles

The basic equation of a circle is

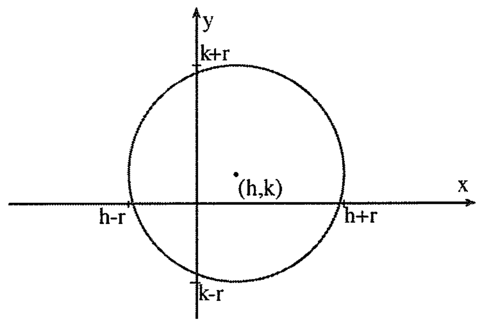
$$x^2 + y^2 = r^2.$$

This circle has centre $(0,0)$ and radius r.
By setting $y = 0$ we find it has x-intercepts at $(r,0)$ and $(-r,0)$.
By setting $x = 0$ we find it has y-intercepts at $(0,r)$ and $(0,-r)$.



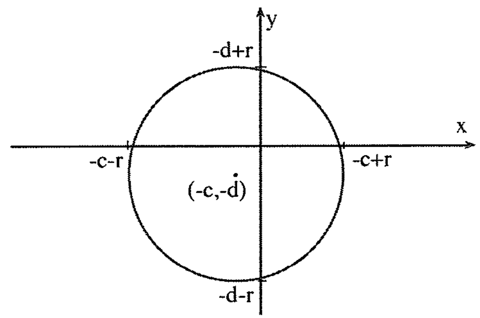
If the circle is shifted h units to the right and k units up, with the radius unchanged, then the centre moves to (h,k) and its equation becomes

$$(x-h)^2 + (y-k)^2 = r^2.$$



Notice that, if h and k are both zero, then the circle has centre at the origin, and we recover our original equation.

Question: What happens if we shift the centre c units to the *left* and d units *down*?



The centre moves to $(-c,-d)$, while the radius remains unchanged.
The equation becomes

$$(x+c)^2 + (y+d)^2 = r^2.$$

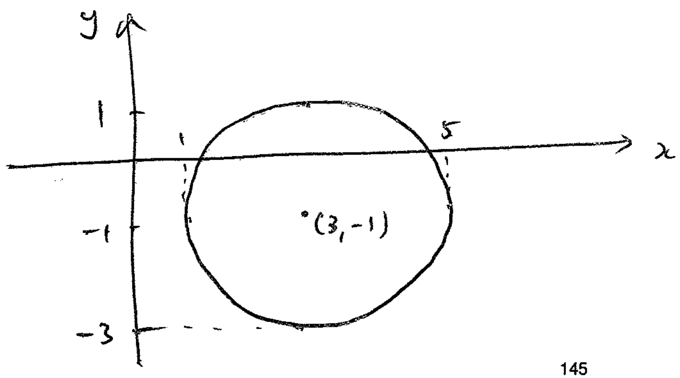
$$(x-(-c))^2 + (y-(-d))^2 = r^2$$

Exercise: Sketch the curve with equation $(x-3)^2+(y+1)^2 = 4$.

$(x-3)^2 + (y+1)^2 = 2^2$

$(x-3)^2 + (y-(-1))^2 = 2^2$

centre $(3,-1)$ radius 2

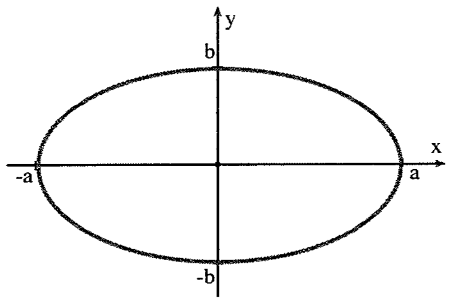


4.1.2 Graphs of ellipses

The basic equation of an ellipse is

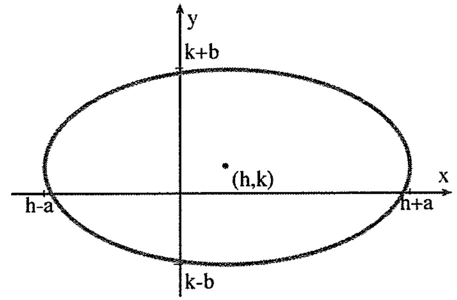
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

This ellipse has centre $(0,0)$.
By setting $y=0$ we find it has x-intercepts at $(a,0)$ and $(-a,0)$.
By setting $x=0$ we find it has y-intercepts at $(0,b)$ and $(0,-b)$.



If the ellipse is shifted h units to the right and k units up, then the centre moves to (h,k) and its equation becomes

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1.$$



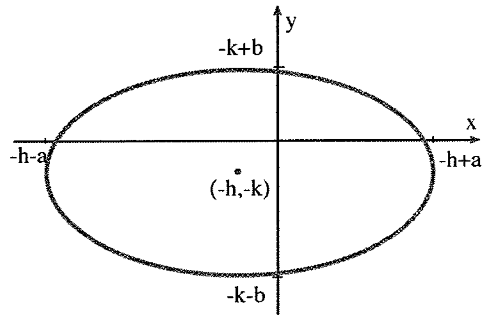
Notice that, if $a=b$ then the equation can be rearranged as

$$(x-h)^2+(y-k)^2=a^2$$

which is just the equation of a circle with centre (h,k) and radius a.

If we instead moved the ellipse h units to the *left* and k units *down*, the centre of the ellipse would move to $(-h,-k)$ and its equation would become

$$\frac{(x+h)^2}{a^2}+\frac{(y+k)^2}{b^2}=1$$

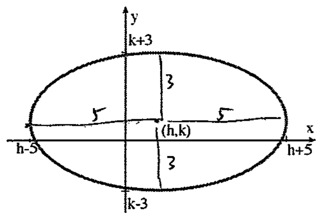


An ellipse does not have a radius. It has a *major axis* and a *minor axis*.

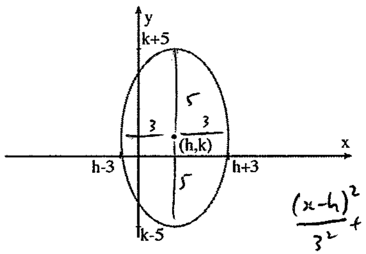
If $a > b$ then the major axis will be parallel to the x-axis and will have length $2a$ whilst the minor axis will be parallel to the y-axis and will have length $2b$.

If $a < b$ then the major axis will be parallel to the y-axis and have length $2b$ whilst the minor axis will be parallel to the x-axis and will have length $2a$.

Confused? Let's see a picture.



$$\frac{(x-h)^2}{5^2} + \frac{(y-k)^2}{3^2} = 1$$



$$\frac{(x-h)^2}{3^2} + \frac{(y-k)^2}{5^2} = 1$$

Example: Sketch the curve with equation $\frac{(x+2)^2}{9} + \frac{(y-4)^2}{64} = 1$.

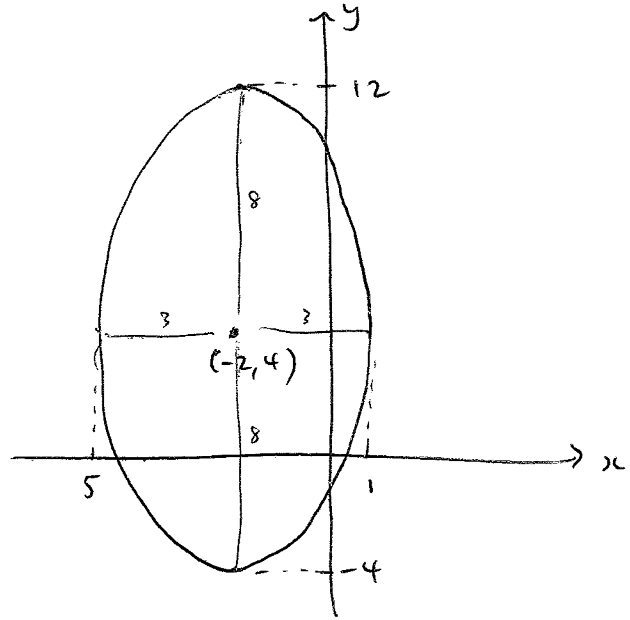
$$\frac{(x+2)^2}{3^2} + \frac{(y-4)^2}{8^2} = 1$$

$$\frac{(x-(-2))^2}{3^2} + \frac{(y-4)^2}{8^2} = 1$$

centre $(-2,4)$

minor axis
length $2\times 3 = 6$
parallel to x-axis

major axis
length $2\times 8 = 16$
parallel to y-axis



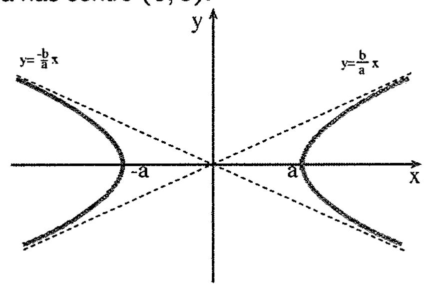
Homework: Sketch the curve with equation $\frac{(x+2)^2}{16} + \frac{(y-4)^2}{9} = 1$.

4.1.3 Graphs of hyperbolae

The basic equation of a hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

This hyperbola has centre $(0,0)$.

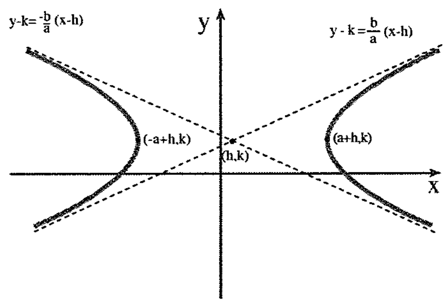


By setting $y = 0$ we find it has x-intercepts at $(-a, 0)$ and $(a, 0)$.
If we set $x = 0$, there are no solutions for y, so no y-intercepts.

There are two (oblique) asymptotes, at $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

If the hyperbola is shifted h units to the left and k units up, its equation becomes

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1\,.$$



The centre is then at (h,k), the *vertices* are $(-a+h,k)$ and $(a+h,k)$ and the asymptotes are

$$y-k=\frac{b}{a}(x-h) \quad \text{and} \quad y-k=-\frac{b}{a}(x-h)\,.$$

Example: Sketch the curve with equation

$$\frac{16(x-1)^2}{144} - \frac{9(y+4)^2}{144} = \frac{144}{144}\,.$$

$$\Rightarrow \quad \frac{(x-1)^2}{9} - \frac{(y+4)^2}{16} = 1$$

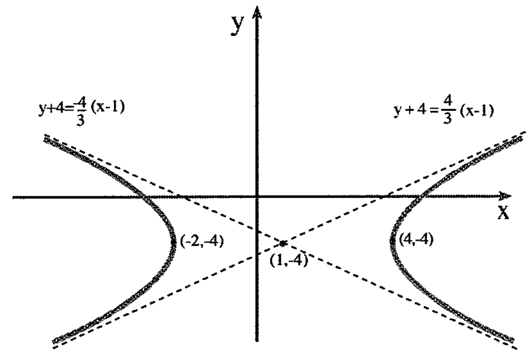
$$\frac{(x-1)^2}{3^2} - \frac{(y-(-4))^2}{4^2} = 1$$

$\Rightarrow$ centre $(1,-4)$

asymptotes $y+4 = \pm\frac{4}{3}(x-1)$

vertices $(-3+1,-4) = (-2,-4)$

$(3+1,-4) = (4,-4)$



Homework: Sketch the curve with equation $\frac{(x-3)^2}{9} - \frac{(y+4)^2}{25} = 1$.

Additional questions

You can now attempt a selection of problems from 9 - 20 in Chapter 11.4 from the textbook (ignore the parts about foci).