Network augmentation for disaster-resilience against geographically correlated failure

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Introduction

Major infrastructure networks are abundant, whether that is telecommunications, transport or power, the reliance on these networks are vastly important in today's fast paced world.

Many of these networks may be susceptible to disconnection through disasters such earthquakes, fires or cyclones. An immediate question arises of how to augment the network (add extra connections) to ensure resilience against said disasters.

We model networks as planar geometric graphs with primary nodes (junctions) and secondary nodes (edge bends), and define disasters as straight horizontal line segments of a fixed length l in this study. Using a simplified disaster model, we demonstrate feasible augmentation algorithms that outperform traditional block-based approaches.

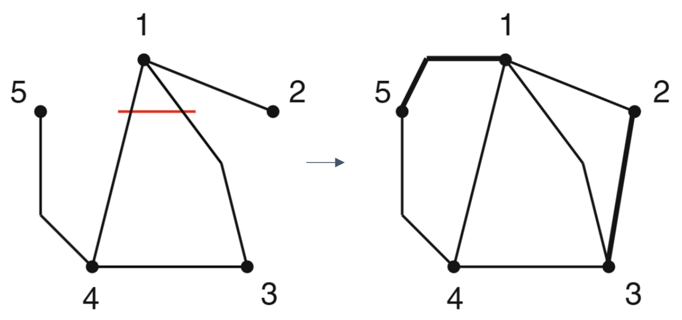

Figure 1. l-resilient augmented network, maintains connectivity despite disaster (red line)

Problem

The problem will be modeled using a geometric network $G = (V, E)$,a connected graph that is embedded in the Cartesian plane. The vertices V are points and the edges E are simple curves in the plane, each connecting two vertices.

- A **disaster** D is a translation of an open horizontal line segment D of length l in the plane. For example; red line if Figure 1
- An **l-cut** of G with topology $\mathcal{G}$ is a cut-set $\mathcal{E}_0$ of $\mathcal{G}$ such that there exists a disaster D of length l which disrupts $E_0(G)$, where $E_0(G)$ is the set of edges of G corresponding to $\mathcal{E}_0$. If $\mathcal{E}_0$ is both an l-cut and a minimal cut-set, it is called a minimal l-cut.
- If $G = (V, E)$ does not contain an l-cut then G is called **l-resilient**,.

The *line segment disaster augmentation problem* (LSDAP) can now be stated as follows. For a given geometric network $G = (V, E)$ and a disaster D of length ℓ, find an ℓ-augmentation $G^* = (V, E \cup E^*)$ of G that minimises $\sum_{e \in E^*} |e|$, where $|e|$ represents the Euclidean length of the embedded edge, defined as the sum of the lengths of the line segments composing the edge.

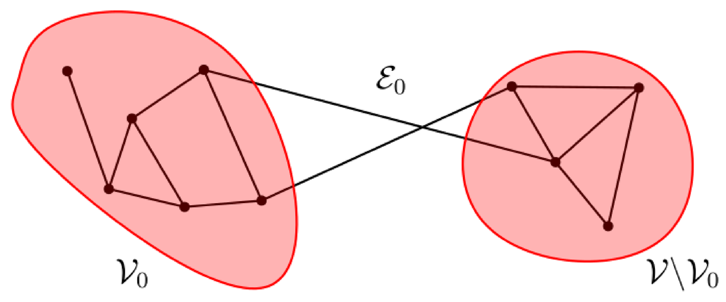

Figure 2. Network with an l-cut, $\mathcal{E}_0$, and it's corresponding disjoint shores,$\mathcal{V}_0$ and $\mathcal{V} \backslash \mathcal{V}_0$

l-blocks

An ℓ-block B of G is a set of vertices of G with the following properties:

- $\forall u, v \in B$, u and v are mutually ℓ-resilient;
- $\forall u \in B$ and $v \notin B$, u and v are not mutually ℓ-resilient.

A graph $\mathcal{G}$ is ℓ-resilient if an only if it contains exactly one ℓ-block.

The concept of ℓ-blocks will be important when finding an algorithm to augment a graph to be ℓ-resilient, the first step of which is identifying bridging edges between mutually exclusive ℓ-blocks that avoid **destruction zones**.

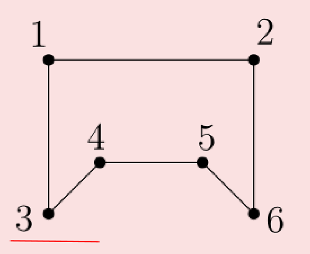

Figure 3. Network with disaster D of width l

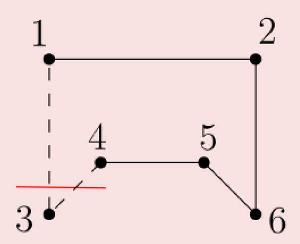

Figure 4. l-cut of$\{(1, 3), (3, 4)\}$

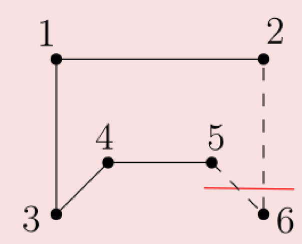

Figure 5. l-cut of$\{(2, 6), (5, 6)\}$

From the above network nodes $\{1, 2, 4, 5\}$for an l-block as they are mutually l-resilient.

Finding l-cuts

Before adding augmentation edges, one must first identify the disconnecting l-cuts with the use of a sweep line algorithm. The algorithm has three subroutines.

- **Activate** runs when the top endpoint of a new segment is encountered by the sweep line. The algorithm pairs this new edge with every other active edge as either being an internal or boundary edge for a potential l-cut.
- **Deactivate** removes a segment from the active set of analysis edges when the bottom endpoint is encountered.
- **Converge** is called when the horizontal distance, $d_h(s_1, s_2)$ decreases as the sweep line passes through. This function will also analyse these two edges as being a potential l-cuts.

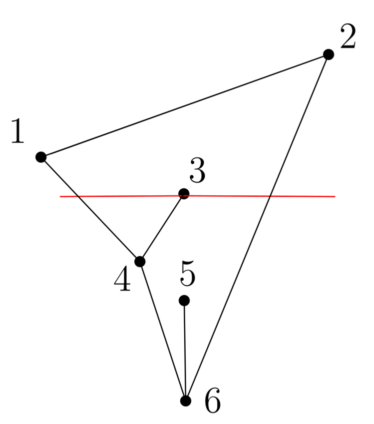

Figure 6. $\{(3, 4)\}$ added to active edges as sweep runs downwards

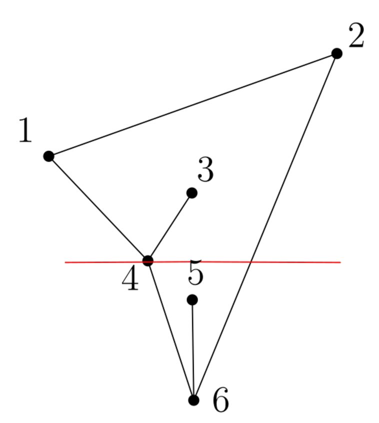

Figure 7. $\{(3, 4)\}$ removed from active edges as sweep runs downwards

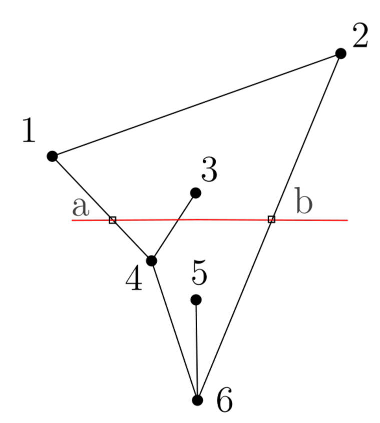

Figure 8. $d(a, b)$ decreases as sweep runs down,$(1, 4)$ and $(2, 6)$ are converging

Augmentation

Each l-cut has an associated **destruction zone**,$\mathcal{D}_{\mathcal{E}_0}$ where in which placing an edge within this zone will result in its destruction also. Intuitively, when augmenting the graph we must avoid these destruction zones.

Let $\mathcal{E}_0$ be an l-cut of $G = (E, V)$ and let $\mathcal{V}_0, \mathcal{V} \backslash \mathcal{V}_0$ be the shores of $\mathcal{E}_0$. Let E_0 and V_0 be the set of embedded edges and vertices of G for $\mathcal{E}_0$ and $\mathcal{V}_0$ respectively. Let p be an embedded polygonal chain,not intersecting the interior of any edge in E, such that one end-node of p lies in V_0 and the other in $\mathcal{V} \backslash V_0$. Then p is said to bridge E_0 and is referred to as a **bridging edge**.

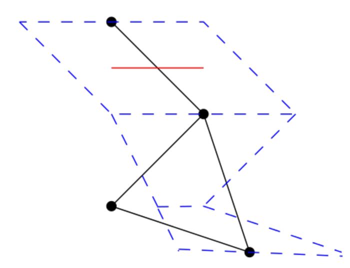

Figure 9. Network with disaster D of width l and the corresponding disaster zones

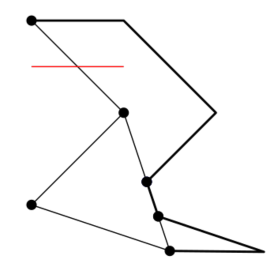

Figure 10. Augmented network with bridging edge between top and bottom nodes, that avoids destruction zones

l-leaves

A set of vertices $V_L \subseteq V$ is called an l-leaf of G if V_L is a minimal shore of a minimal l-cut of G; that is, for every other minimal l-cut of G with shores V_1, V_2, we have $V_1 \nsubseteq V_L$ and $V_2 \nsubseteq V_L$.

This means, for any l-cut of a network G, the nodes within the leaf cannot be separated. A network is l-resilient if and only if it contains no l-leaves.

Finding shortest chain

Overlaying the faces created by G and the disaster zones,$\mathcal{D}_{\mathcal{E}_0}$, of each l-cut of G, the problem becomes an obstacle avoidance one, which has been extensively studied.

- A subdivision A of the plane is created by overlaying G and the boundaries of $\mathcal{D}_{\mathcal{E}_0}$ corresponding to each of the l-cuts found.
- A visibility graph is created designed to avoid every $\mathcal{D}_{\mathcal{E}_0}$ by treating them as obstacles
- A shortest path from V_1 to V_2 (shores of each) using the visibility edges is computed if one exists.

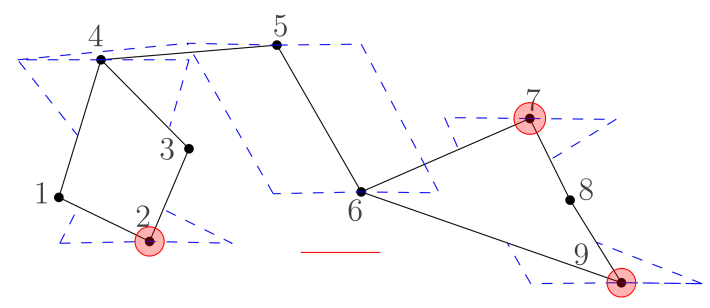

Figure 11. Network with blue destruction zones,$\mathcal{D}_{\mathcal{E}_0}$ and red highlighted l-leaves

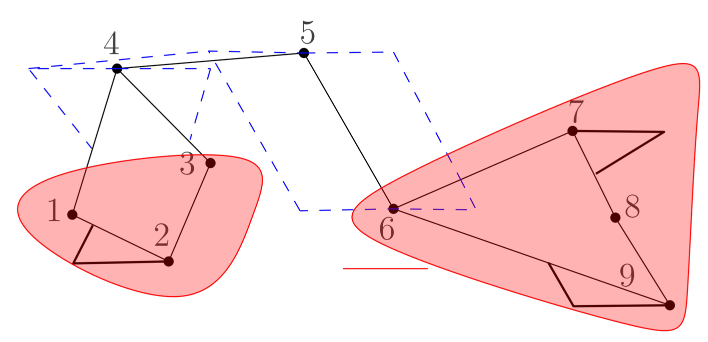

Figure 12. First step of expanding l-leaves, whilst avoiding $\mathcal{D}_{\mathcal{E}_0}$

Heuristics and feasibility

The l-leaves can be joined in a few different ways, testing these heuristic schemes against each other sprouts questions regarding feasibility of disaster augmentation.

For example, we can choose to restrict the end-nodes u, v of p so that u lies inside an l-leaf, say L, of the current network topology and v lies outside of L. There are instances however, where this scheme can fail to find a feasible connection.

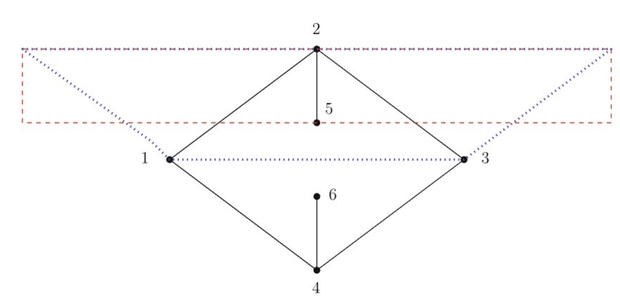

Figure 13. An example where the LeafOrigin-StrongConnection scheme fails to find a feasible solution

Conclusions and future work

This study provided the first formal framework for the study of disaster resilience for polygonal networks. Many of the test cases yielded positive results such as the augmented network below.

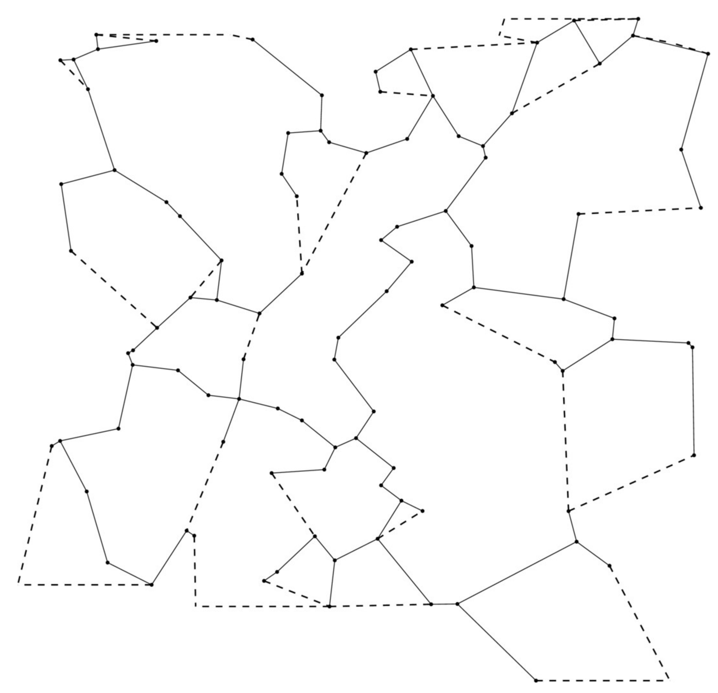

Figure 14. Augmentation of a network with 50 vertices. The input network is a minimum spanning tree (solid lines). The network shows the augmented network using the scheme *LeafCombination(3)-WeakConnection*, where the added edges are drawn as dashed lines.

The toolbox of concepts developed from this simple disaster type can be expanded to more complex regions in the plane.

Acknowledgments

[1] Nicolau Andrés-Thió, Marcus Brazil, Charl Ras, and Doreen Thomas.
Network augmentation for disaster-resilience against geographically correlated failure.
Networks, 81(4):419–444, 2023.

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