When do wireless network signals appear Poisson?

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Motivation

Let's say a mobile user sitting in a random environment and receives signals from many transmitters placed according to some deterministic or random process.

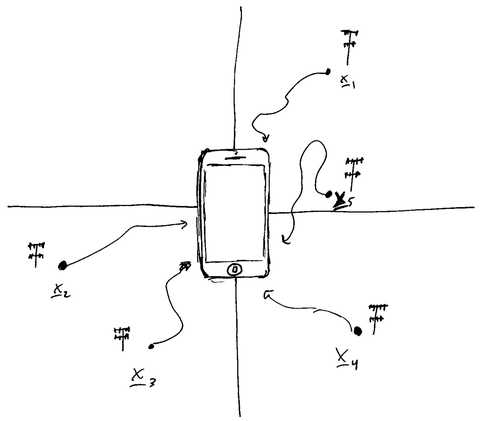

Fig. 1: An example of receiver and several transmitters

We are interested in the signal strength received by the user under various conditions of the environment. The signal strengths received by the user are typically affected by its distance to the transmitters and by physical fading effects which are often modeled as random variables.

SSID	BSSID	Signal	Last seen
AWESOME99	00:60:64:94:33:DA	-37	now
Our Network @ Cardigan	C8:BE:19:8F:6B:FC	-64	now
OPTUSA2290FE	00:1E:2A:22:90:FE	-78	now
TPG-Z45T	F4:DC:F9:10:22:58	-81	now
ii1EDE44primary	78:A0:51:1E:DE:45	-86	now
Norwegian Jammers	6C:19:8F:61:F9:02	-87	now
Bigpond1016	40:F2:01:BC:B9:28	-	3min 59s ago
BigPondD9DC65	08:76:FF:D9:DC:65	-	3min 54s ago
car-home	00:60:64:96:DA:A4	-	17s ago
ii17A7F0primary	78:A0:51:17:A7:F1	-	1min 18s ago
NETGEAR49	28:C6:8E:5A:64:4E	-	4min 5s ago

Fig. 2: An example of WIFI signal strength

We aim to study the distribution of the point process of signal strengths experienced by a typical user.

Acknowledgements

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References

[1] H. Keeler, N. Ross, and A. Xia. "When do wireless network signals appear Poisson?" In: *Bernoulli* 24.3 (2018), pp. 1973–1994. DOI: 10.3150/16-BEJ917. URL: https://doi.org/10.3150/16-BEJ917.

[2] N. Ross. "Fundamentals of Stein's method". In: *Probability Surveys* 8.none (2011), pp. 210–293. DOI: 10.1214/11-PS182. URL: https://doi.org/10.1214/11-PS182.

Model

The model can differ depending on different types of networks. The one studied here is the most standard model, where we have a receiver placed at the origin of a d dimensional space $\mathbb{R}^d$, and transmitters placed at locations $\Xi = \{X_i : i \in \mathbb{N}\} \subseteq \mathbb{R}^d \backslash \{0\}$ as in figure 1. We are interested in the signal strength P_i received by the observer from X_i. A standard assumption is that

- the signal strength received is inversely proportional to the distance between the transmitter and the observer.
  - the path loss function $l(x) = C|x|^{-\beta}$ for some positive constants C and β.
- Another factor that affects the signal strength is known as the propagation effect.
  - multipath fading, where the signal strength is reduced due to signals taking multiple paths and colliding with each other.
  - shadow fading, where signal strength is affected due to signals colliding with large obstacles such as buildings.

We can incorporate these effects into the model via a sequence of independent and identically distributed random variables $S, S_1, S_2, ...$, and the signal strength from transmitter X_i is then given by

$$P_i = S_i l(X_i) =: \frac{S_i}{g(X_i)}$$

where $g(x) = 1/l(x)$.
We are interested in the distribution of the process on $\mathbb{R}^+$

$$\Pi := \{P_i\}_{i \in \mathbb{N}}$$

and various functions of it. For example, the signal-to-interference ratios $\{P_i/(\sum_j P_j - P_i)\}_{i \in \mathbb{N}}$. In wireless networks, it is possible to detect signals from multiple sources, but the receiver typically only wants one of them. The desired signal is the P_i on the numerator and the unwanted signals are in the denominator of the ratio. The signal from a particular source X_i is detectable if and only if $SIR(X_i)$ exceeds the so-called SIR threshold $\tau > 0$. The probability $P(SIR(X_i) > \tau)$ is usually referred to as the coverage probability, is the one we are interested in.
Researchers usually study the inverse of the signal strength instead of the original one since there tend to be many weak signals that cause the approximating Point process to have a singularity at zero. That is

$$N = \{1/P_i\}_{i \in \mathbb{N}} = \{g(X_i)/S_i\}_{i \in \mathbb{N}},$$

and results about N can be easily translated to results for Π.

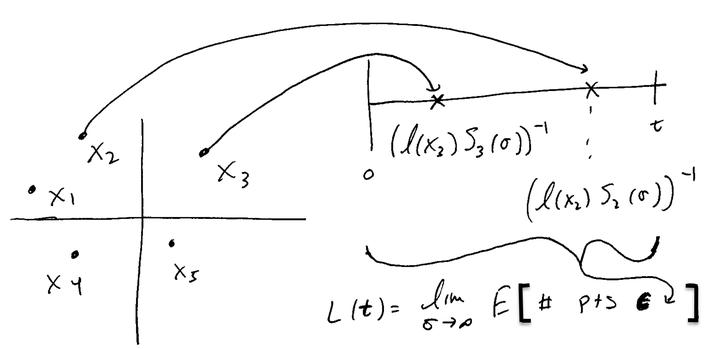

Fig. 3: An example of propagation process experienced by the receiver

The process N has been called the (independently) propagation (loss) process or path loss (with fading) process generated by S, g, and Ξ.

Method

Before showing the main results, we first introduce some important definitions and methods.

Definition 1: A sequence $\{X_n\}_{n \in \mathbb{N}}$ of random variables converges in probability towards the random variable X if for all $\epsilon > 0$,

$$\lim_{n \to \infty} P(|X_n - x| > \epsilon) = 0.$$

We also need the concept of convergence in distribution of a point process, but due to technicality, we instead introduce convergence in distribution of a random variable as the ideas are similar.
Definition 2: A sequence of real-valued random variables $X_1, X_2, ...$, with cumulative distribution functions $F_1, F_2, ...$, is said to converge in distribution to a random variable X with cumulative distribution function F if

$$\lim_{n \to \infty} F_n(x) = F(x),$$

for every number $x \in \mathbb{R}$ at which F is continuous.

The main tool for proving the results is called Stein's method (for backgrounds, see [2] for a nice survey on this topic). Stein's method is a powerful technique in probability theory that bounds the distance of two probability distributions concerning some probability metric. The choice of the metric depends on the nature of the approximating distribution. In the case of Poisson point process approximation, the most widely used is the total variation metric which bounds the maximum difference in probabilities between two probability distributions.

Definition 3: The total variation distance between the distribution of the random process N, and the Poisson point process Z is given by

$$d_{TV}(\mathcal{L}(N), \mathcal{L}(Z)) := \sup_{A \subset \mathcal{B}(\mathcal{D})} |P(N \in A) - P(Z \in A)|, \tag{1}$$

where $\mathcal{L}(X)$ denotes the distribution of X.

The general idea of proving convergence using Stein's method is as follows. we aim to obtain a bound on the total variation distance between the distributions of N and Z so that the bound goes to 0 at some suitable rate that is sufficient to prove convergence in distribution.

Result

One of the results shows that the distribution of N converges to a Poisson point process under some conditions [1].

Theorem [1]: If $\{X_i\}_{i \in \mathbb{N}}$ and $\{l(X_i)\} \subset (0, \infty)$ are both locally finite. For each $\sigma > 0, S(\sigma), S_1(\sigma), ...$ are independent and identically distributed positive random variables satisfying
(i) $S(\sigma) \xrightarrow{P} 0$ as $\sigma \to \infty$.
(ii) There is a function $L : (0, \infty) \to (0, \infty)$ with

$$\mathbb{E}[\{i : N_i(\sigma) \leq t\}] \to L(t),$$

as $\sigma \to \infty$ for all continuity points of L.
Then the point process $\{N_i(\sigma)\}_{i \in \mathbb{N}}$ converges in distribution to a Poisson point process on $(0, \infty)$ with mean measure $L(t)$.

Referring to Figure 3, the first condition $S(\sigma) \xrightarrow{P} 0$ gives that the chance the inverse signal power from a given transmitter falls in an interval $(0, t)$ tends to zero. The second condition ensures the limit is not degenerate, which means that the mean measure is positive and finite.