Melbourne University Mathematics Society
Problem Solving Competition

Competition Rules

- Teams must have either 3 or 4 competitors.
- Teams must be either all students or all teachers. No mixed teams.
- No calculators.
- The competition will consist of four rounds, each lasting 13 minutes
- In each round there are five questions, but only your best three questions will contribute to your score. The questions have 1, 2, 3, 4, and 5 points allocated respectively.
- You will not lose points for incorrect answers
- Prizes will be awarded to the top three school teams and top teachers' team
- You must have fun!

Round One

1. What is the highest possible score in this competition?

2. Solve this system of equations:

$$x - 2y + 3z = 7$$
$$2x + y + z = 4$$
$$-2x + z = -3$$

3. Everyone at a party with 25 people has to shake hands with everyone else. How many handshakes will occur?

4. Find a 3-digit perfect square such that the first and last digit add to the middle digit.

5. How many non-congruent strictly isosceles (not equilateral) triangles can be constructed using only edges of length 3cm, 7cm or 10cm?

Round Two

1. Three cards are drawn at random with replacement from a standard deck (without jokers). What is the probability that at least two of the three cards are red?

2. Madeleine owns some sheep and ducks, who together have 28 heads and 82 legs. How many ducks does Madeleine have?

3. If 3x−y=12, what is the value of $\frac{8^x}{2^y}$?

4. For a positive integer n, let f(n) be the smallest positive integer with exactly n factors (including itself). So f(1) = 1, f(10) = 48 and so on. What is f(f(f(f(2))))?

5. How many four-digit numbers have no repeat digits, do not contain zero, and have a sum of digits equal to 28?

Round Three

1. Samoil walks 5 metres west. He turns 90° right and walks 6 metres. He then turns west and walks 7 metres. How far is Samoil from his starting point?

2. When I was 31 years old, my son was 7. Now he is half as old as me, how old am I?

3. Simplify i^{231} where i is the imaginary unit and is defined as:
$i = \sqrt{-1}$

4. It is pi day and Emma only has one pie! The first person is allocated ⅓ of the pie, the second person ¼ of the pie, the next person ⅕ of the pie and so on. How many people will get their full allocation of pie?

5. The notation a_b means that a is written in base b. For example, $21_3=7$. The number 101_b is one less than a perfect cube. What is b?

Round Four

1. The 2 solutions of the equation, $x^2 - 7x + 12 = 0$ are α and β. What is $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$?

2. What is the 10th number in this sequence: 1,3,6,10,15,...?

3. How many ways can you arrange the letters in 'MATHEMATICS'? (You do not need to simplify the answer)

4. The radius of a right cylinder is increasing at a rate of 8 mm/s and the height is decreasing at a rate of 15 mm/s. Find the rate at which the volume is changing in cm^3/s when the radius is 40 mm and the height is 150 mm?

5. McDonuts are sold in packs of 5, 11, and 18. What is the largest number of donuts I cannot buy without splitting packs?